

1984

## Two essays on money supply growth, inflation, and interest rates

Masoud Moghaddam  
*Iowa State University*

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TWO ESSAYS ON MONEY SUPPLY GROWTH, INFLATION, AND INTEREST  
RATES

*Iowa State University*

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Two essays on money supply growth, inflation, and  
interest rates

by

Masoud Moghaddam

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## GENERAL INTRODUCTION

This dissertation is composed of two studies of how the interest rate responds to inflation and to the growth rate of the money supply: part one deals with the impact of unanticipated money stock growth ( $UM_t$ ) on market interest rates. It also sheds some light on the question of whether anticipated money stock growth ( $AM_t$ ) affects interest rates. The efficacy of ( $UM_t$ ) stems from the idea of rational expectations, which has created a revolution in macroeconomics over the past decade.

If market participants utilize all available information and revise it whenever needed, then expectations are said to be formed rationally. Rationality in turn implies that individuals do not make systematic forecasting errors, and on the average, guess correctly. In such an environment, all anticipated policies of the central bank are incorporated in the general public's expectations, thereby they do not have real effects. To the contrary, if the central bank's policies come as a surprise to the public, then there will be real effects in the economy.

To probe the neutrality and rationality underlying anticipated and unanticipated changes in the money supply growth rate, data from bond markets are useful, because interest rates on bonds are quite sensitive to a change in the rate of growth of the money supply. Therefore, assuming that bond markets are relatively efficient markets, a change in the rate of growth of the money supply, whose primary impact is on the expected rate of inflation, will be fully incorporated in the market rate of interest. In an efficient asset market, an increase in ( $UM_t$ ) creates a domi-

nant liquidity effect which puts downward pressure on the short- and medium-term interest rates. Lower interest rates stimulate investment and expenditures through the transmission mechanism. However, an increase in  $(AM_t)$  increases the expected rate of inflation and puts upward pressure on the nominal interest rates. Hence,  $(AM_t)$  has a dominant price expectational effect with no impact on the real variables.

Part two is concerned with the search for empirical support for the Darby effect, which has been a subject matter for many studies since 1975. The Darby effect grew from the Fisherian hypothesis, in which there is a one-to-one relationship between a change in the expected rate of inflation  $(\pi_t^e)$  and changes in nominal interest rates. In an econometric sense, the regression coefficient on  $(\pi_t^e)$  would be one. The Fisherian belief is applicable in an economy without income taxes. Since interest income is subject to income taxes, the Darby effect implies that the nominal interest rate must increase by more than the expected rate of inflation. Otherwise, lenders will not be compensated for both interest income taxes and losses due to inflation.

The results of empirical testing of the Darby effect have been mixed. Nevertheless, one study completed in 1983, concluded that over the sample period 1952-79, the Darby effect did exist. The primary purpose of part two is to reestimate and to extend the Darby model. Also, it is demonstrated that the Darby effect reported in the above study is heavily affected by impure autocorrelation, and as such, is doubtful.

PART ONE

INTEREST RATES AND UNANTICIPATED  
MONEY GROWTH IN THE CONTEXT  
OF EFFICIENT MARKETS

## SECTION I. INTRODUCTION

The neutrality of money and the natural rate hypothesis under rational expectations have been the subject matter of a number of economic studies over the past decade. For example, Barro (1978 and 1981), Gordon (1979), Barro and Rush (1980), and Mishkin (1982) have tried to test the natural rate proposition. Barro-Rush's work has been criticized on several different bases. Nevertheless, their decomposition of the money supply stock into anticipated and unanticipated components has drawn much attention.

Barro-Rush's empirical test results imply that on the one hand, anticipated money supply growth does not matter, because it leaves the real variables (output, unemployment) unaffected at their natural rate. On the other hand, the effectiveness of unanticipated monetary policies stems from the fact that economic agents' information regarding the general price level on the global market (unlike the local market) is incomplete. Therefore, an unanticipated change in the money supply growth has an effect on the real variables, because it confuses economic agents between a change in the general price level and changes in relative prices. The outcome of such confusion is that if prices on the local market rise without complete information, then market participants react as if their relative prices are rising, and thereby act accordingly.

In a related study, Mishkin concluded that Barro-Rush's tests are invalid due to the absence of enough lags in the model (see Mishkin 1982 and 1983). Furthermore, Mishkin, in the context of nonlinear joint esti-

mation and the statistical likelihood ratio test with 20-quarter lags in the model, showed that an anticipated money supply growth is as important as unanticipated money growth. Therefore, he rejected the neutrality proposition underlying the macroeconomic rational expectations hypothesis.

Assuming that the Barro-Rush's time series data of anticipated and unanticipated money supply growth are correct, the primary purpose of this study is to test the response of two short-term and two long-term interest rates to anticipated and unanticipated money supply growth. The rationale behind such a test, is to determine whether the anticipated and unanticipated money supply growth rates have different directional impacts on interest rates.

This test is of particular interest for two reasons. One, to investigate whether Barro-Rush's quarterly and yearly time series data are basically consistent with the notion of rational expectations in asset markets. If so, an anticipated increase in money supply growth causes an anticipated increase in the rate of inflation, which in turn increases nominal interest rates. Correspondingly, an unanticipated increase in money supply growth creates liquidity effects, which causes interest rates to fall. Two, to shed some light on the importance of anticipated monetary policies in different bond markets.

The organization of the study is as follows. Section II deals with a review of related literature. In Section III, interest rate models are constructed, and in Section IV empirical tests and results are discussed. Then in Section V, the conclusions of the study are presented.

## SECTION II. A REVIEW OF RELATED LITERATURE

## Rational Expectations and the Natural Rate Hypothesis

According to the doctrine of rational expectations, individuals utilize all available information in forming expectations of future relevant economic variables and revise their information sets whenever needed. Furthermore, individuals are forward-looking when forming their expectations and do not make systematic errors.

In the absence of perfect foresight, rational expectations proponents argue that individuals should not only form their expectations according to past values of the variable, but also to the past value of all relevant information as well, particularly in a simultaneous equation model in which the interaction among variables becomes very crucial. In such an environment, the true expectations which are based on the probability of occurrence, are the mathematical expectations, which are conditional on all presently available and expected future information.

If on the average, the economic agent is not wrong in regards to future expectations, then the structure of the model and the probability attached to the error term is assumed to be known. In essence, rational expectations are formed as an endogenous part of a model, by assuming that in the absence of systematic forecasting errors, individuals who on the average guess correctly, act as though they understand the systematic portion of the model, as well as the structure and the expected value of the error term.



The new classical macro-economists believe that disequilibrium in the labor and commodity market disappears quickly - due to the fact that wages and prices are assumed to be fully flexible. Consequently, output and employment remain at their natural rates. Therefore, an anticipated increase in the rate of growth of the money supply would leave real money balances and real wages constant at the full-employment level (the natural level) by increasing nominal wages and prices in order to absorb the shock created by the expansionary monetary policies.

On the issue of money neutrality, coupled with the long-run debate dealing with the trade-offs between unemployment and inflation - especially in regards to the question of which one is socially desirable, economic theory seems to reach an impasse. Hence, the theory of rational expectations seems to present the ultimate solution to the inflation-unemployment trade-off dilemma. Models developed under rational expectations by Lucas (1972), Sargent (1973), Sargent and Wallace (1975), Barro (1976), and Barro and Rush (1980), reached a revolutionary conclusion. That is, monetary policies conducted in a systematic manner do not have real effects, even in the short-run.

The philosophy of rational expectations is in agreement with Friedman (1968), who argued for a more predictable monetary policy, rather than other stabilization policies. Likewise, it is in line with the ineffectiveness of monetary policies in the long-run, in which the Phillips curve is perfectly vertical. Also, it justifies the Fisherian hypothesis (1930) - that there is a direct relationship between a rise in the expected rate

of inflation and an increase in the nominal interest rate (for more information, see Begg 1982).

#### Rational Expectations and the Role of Monetary Policies

Economic agents have very limited knowledge of the markets with which they are not in direct contact, and so cannot observe all prices on the different markets at the same time. This is either due to the cost of gathering such information, or is due to a lack of interest, coupled with ignorance of market participants.

In the absence of perfect foresight, given the limited amount of information available, a sudden change in the money supply growth would also necessitate the reassessment of the informational set. Hence, if the monetary authority increases the rate of growth of the money supply and subsequently increases the general price level, then individuals would consider this to be an increase in the price of their goods (in their market basket), relative to the price of other goods which they cannot observe. Consequently, producers produce more and work more. So, mistakenly, economic agents think that their nominal wages and prices have risen, relative to others with whom they cannot communicate. By the same token, surprise changes in the growth rate of the money supply can be held responsible for business fluctuations.

Suppose that there are  $(N)$  markets producing a commodity  $(Y_t)$ . At any point of time, only one market (say  $Z$ )<sup>1</sup> can be visited by a consumer

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<sup>1</sup> $(Z)$  can also be viewed as different goods in the local market.

or a producer, but the market participants can move across the markets freely. Assume also that  $P_t(Z)$  is the price of a market basket of commodities in location (Z), and  $P_{(t)}$  is the general price level at time (t). If  $P_t(Z)$  differs from  $P_{(t)}$ , then there will be entry and exit from the market (Z) by producers and workers. The process continues to a point where the local price is quite close to the general price level, which is the average price of different market prices, i.e., the expected value of  $P_t(Z)$  is always  $P_{(t)}$ .

The production function in market (Z) for commodity ( $Y_t$ ) is

$$Y_t(Z) = F[L_t(Z), K_{t-1}(Z)]; F'_L = \frac{\partial F}{\partial L_t} > 0, F'_{K_{t-1}} = \frac{\partial F}{\partial K_{t-1}} > 0, \frac{\partial F'_L}{\partial L_t^2} < 0 \quad (2-1)$$

and  $\frac{\partial F'_{K_{t-1}}}{\partial K_{t-1}^2} < 0$

where  $L_t$  is the amount of work performed by the labor force at time (t), and  $K_{t-1}$  is one period lagged capital stocks. An increase in the relative price  $P_t(Z)/P_{(t)}$ , increases  $Y_t(Z)$  along with increasing the incentive to work, but capital stock remains unchanged. However, an increase in the expected relative price  $P_{t+1}(Z)/P_{t+1}$  in market (Z) would stimulate capital accumulation at time (t).

Market (Z) is in equilibrium when the total supply  $Y_t^S(Z)$  is equal to the total demand  $Y_t^D(Z)$ . For the average market (Z) which has not experienced any drastic changes in  $Y_t^S(Z)$  and  $Y_t^D(Z)$ , the market clearing relative price equals unity. That is, on the average market, the local market price is equal to, or reasonably very close to, the average price level.

Assume that there is perfect information and that the money supply increases once-and-for-all. The general price level increases and economic agents spend extra income (money) on different goods and services. The excess demand increases the price of these goods and services in different market locations in such a manner that the relative prices remain constant. As such, a change in the money supply growth is incapable of changing real output and employment, as well as relative prices, i.e.,  $P_t(Z)$  and  $P_{(t)}$  will increase in proportion to the increase in the money supply.

It is obvious that both producers and consumers have relatively good information concerning prices with which they have dealt, or wages that they have paid/received. In other words, individuals in market (Z) have more information concerning  $P_t(Z)$  than the general (average) price level  $P_{(t)}$ . Therefore, as far as the general price level is concerned, individuals have some prior expected price ( $P_t^e$ ) in mind. Although the local price level in each market is easy to observe, it is relatively difficult to assess how the local market price stands relative to other market prices or the average price level.

Assuming rationality in forming expectations, economic agents must construct their expectations in regards to  $P_{(t)}$ . In this regard, there are two different processes. One, the process of forming ex-ante expectations, which are formed given all past behaviors of output, real interest rates, money supply, and other relevant economic variables without considering the local price  $P_t(Z)$ . Two, ex-post expectations ( $P_t^e, Z$ ), which are

related to revising and correcting expectations whenever the structural model needs to be reassessed after  $P_t(Z)$  has been taken into account.

$(P_t^e, Z)$  depends upon two sets of prices:  $P_t(Z)$  and the prior expectations price ( $P_t^e$ ). The local price  $P_t(Z)$  in turn is a function of how different the price is, from one local market to another. ( $P_t^e$ ), however, is mainly dependent upon how volatile the economy in that market has been, prior to forming expectations. Thus,  $(P_t^e, Z)$  can be summarized in the following equation (see Barro 1983, page 471).

$$(P_t^e, Z) = \theta P_t(Z) + (1-\theta)P_t^e, \quad 0 < \theta < 1 \quad (2-2)$$

The weight ( $\theta$ ) on each parameter is determined by the functioning of  $P_t(Z)$  and  $P_t^e$ . That is, if the local price across the markets do not differ drastically, then ( $\theta$ ) is greater and  $(1-\theta)$  is smaller. Conversely, if the economy has not experienced any severe change, then more weight will be placed on  $P_t^e$ . Given the value of ( $\theta$ ), the ex-post price expectations will also determine the perceived price ratio  $P_t(Z)/(P_t^e, Z)$  by consumers and producers.

An increase in  $P_t(Z)/P_t^e$ , with other things remaining the same, would lead to an increase in the  $P_t(Z)/(P_t^e, Z)$ . However, the rise in the ex-post price expectation is always a fraction ( $\theta$ ) of the increase in the local price, so the rise in the perceived relative price is less than  $P_t(Z)/P_t^e$ . Then, in the absence of drastic external and internal shocks on the market, the overall equilibrium occurs when the price ratio  $P_t(Z)/P_t^e$  and the perceived relative price equal unity. This in turn indicates that  $P_t(Z)$  is also equal to the prior (ex-ante) expected price ( $P_t^e$ ).

In the real world in which there is not perfect information, suppose that the money supply, once-and-for-all, has increased. Due to incomplete information, this increase in the money supply growth comes as a surprise, because  $(P_t^e)$  has already been formed, and cannot include the rise in the money supply. Assuming that the market participants spend extra cash balances on different goods locally,  $P_t(Z)$  increases, so  $P_t(Z)/P_t^e$  rises, along with the perceived price ratio. But the participants in the local market think that the relative prices on the market have risen. Of course, this is nothing more than confusion, because if  $P_t(Z)$  rises, then there will be an increase in  $P_t^e$ , the forecasted price ratio, and the perceived price ratio as well. Indeed, the market participant underestimates the general price level increase, and overestimates the increase in the relative prices. Hence,  $Y^s(Z)$  increases, while  $Y^d(Z)$  decreases.

A typical economic agent thinks that he/she is located in a market where the relative price is high. In actuality, this is not so, because the average local prices  $P_t(Z)$  are always equal to the general price level  $P_t$ . The unexpected increase in the money supply growth and the price level, coupled with the lack of direct information about either the average price level or the quantity of money, confuses the typical market participant between a rise in the general price level and a rise in the relative price ratio in the local market. Therefore, the confusion created by an unanticipated change in the rate of growth of the money supply ( $UM_t$ ), has an effect on the real variables in the economy.

An increase in  $(UM_t)$  increases  $P_t(Z)$  and producers interpret this as if the relative prices have increased, so they work and produce more. Also, higher relative prices would increase the expected relative prices, which means investment is stimulated. Hence, due to an unanticipated money supply growth, all these real variables (real output, investment, and relative prices) are affected, so that the  $(UM_t)$  is not neutral and is capable of creating business cycles.

If the changes in the money supply and the general price level are anticipated by market participants, then there will be no real effect, because the prior expectations price  $(P_t^e)$  has already taken the change in  $P_t(Z)$  into account, i.e.,  $(P_t^e)$  and  $P_t(Z)$  grow at the same rate. An anticipated money supply growth does not create any confusion, thereby real output, employment, and relative prices remain constant, so that anticipated monetary policies are neutral.

In the case of unanticipated monetary policies, although the confusion may persist for a short period of time (since under rational expectations, individuals are supposed to learn from their past mistakes), the impact on the real variables is quite long lasting. The confusion caused by a surprise increase in the money supply growth not only increases the perceived relative price, but also the expected relative price - which means that investment demand will increase. After plants and equipment are in the process of production, even if the confusion is fully understood, the investment cannot be stopped immediately. Thus, the confusion has long-run effects on the real variables.

Rational expectations also have a special implication for the monetary authorities, for if the policies of the central bank during recessionary or expansionary periods are incorporated into the general public's expectations, then the central bank's monetary policies have no real effects. Hence, the monetary authorities' policies must come as a surprise, if they are aimed at changing real variables. The anticipated policies of the central bank do not create confusion between general and relative prices, so real variables are left unchanged. This sometimes is referred to as "the irrelevance result for systematic monetary policy."

#### Rational Expectations and Unanticipated Money Growth

It is conceivable to decompose money supply growth between anticipated ( $AM_t$ ) and unanticipated ( $UM_t$ ) portions, and then to test the neutrality and rationality propositions underlying macroeconomic models (see for example Barro and Rush 1980).

Barro and Rush used a two-step regression procedure which is explained here in general terms for the sake of brevity. Step one involves the estimation of a linear forecasting model by the ordinary least squares (OLS), such as

$$X_t = Z_{t-1} \gamma + U_t^* \quad (2-3)$$

where ( $X_t$ ) is a stimulating aggregate demand policy, for example money supply growth.  $Z_{t-1}$  is a vector of variables used to forecast ( $X_t$ ). These variables are assumed to be known at ( $t-1$ ), ( $t-2$ ), . . . Also, ( $\gamma$ ) is a vector of coefficients, and ( $U_t^*$ ) is an unserially correlated error



term. The anticipated money supply growth ( $AM_t$ ) is estimated on the basis of equation (2-3). Then, the estimated residual of the above model,

i.e.,

$$\hat{U}_t^* = X_t - Z_{t-1} \gamma \quad (2-4)$$

implies the unanticipated money supply growth ( $UM_t$ ). Step two includes the residual in model (2-4) as an aggregate demand variable of a model in the following form:

$$Y_t = \tilde{Y}_t + \sum_{i=0}^n \beta_i U_{t-i}^* + \varepsilon_t \quad (2-5)$$

where ( $Y_t$ ) is unemployment or real output and ( $\tilde{Y}_t$ ) is natural unemployment, or natural real output at time ( $t$ ). ( $\beta_i$ ) are the regression coefficients and ( $\varepsilon_t$ ) is a classical white noise error term. Testing of the neutrality proposition requires the inclusion of contemporaneous and lagged values of ( $X_t$ ) in equation (2.5), that is

$$Y_t = \tilde{Y}_t + \sum_{i=0}^n \beta_i' U_{t-i}^* + \sum_{i=0}^n \delta_i X_{t-i} + \varepsilon_t \quad (2-6)$$

where ( $\beta_i'$ ) and ( $\delta_i$ ) are two vectors of coefficients. The ordinary F-test is used to determine whether ( $\delta_i$ ) is different from zero.

Although Barro and Rush's statistical results give the real output and unemployment equations more empirical support, the price equation was doubtful, which may have been caused by misspecification of the model. Barro and Rush used two sets of yearly and quarterly data for the sample period 1941-1977. Using quarterly data makes the problem of serial correlation more serious than does the yearly data. Nevertheless, it allows for lagged responses of unemployment and output to money supply shocks.

Over the sample period, 1949-1977, the unemployment rate was inversely and significantly related to unanticipated money supply growth and its

two-period lag. Over 1946-1977, the real output (GNP in 1972 dollars) was positively and significantly correlated with  $(UM_t)$  and lagged  $(UM_t)$ . From 1948-1977, the price level turned out to be positively related to the money supply, and inversely correlated with  $(UM_t)$  through  $(UM_{t-5})$ .

The two-step regression procedure makes  $(UM_t)$  orthogonal to the explanatory variables in the money supply growth equation. To avoid such statistical problems, a joint estimation was applied (see Barro and Rush 1980, pages 27-28). For the same data set and sample period, joint estimation improved the regression coefficients in unemployment and real output equations. However, in attempting a joint test of all the equations including the price equation, the statistical results deteriorated, which might be partially due to the bi-directional effect of the price level on the money supply growth rate.

Barro and Rush reestimated the money supply growth equation with quarterly data and the sample period 1941:I - 1978:I, in which they regressed the growth rate of  $M_1$  on its 6 quarter lags,  $FEDV_t$  (the deviation of current government expenditures from the normal level), and the unemployment rate - along with its three-quarter lagged values. As with the yearly time series data, the residuals of the money supply growth equation were assumed to be the unanticipated money supply growth series. To avoid the problem of serial correlation with quarterly data, a second-order autoregressive procedure was employed. Consequently, the regression coefficients in output and unemployment equations indicated that the sign pattern was the same as for yearly data, but the serial correlation de-

creased the contemporaneous coefficients on ( $UM_t$ ) and also shortened its lag response.

Overall, the quarterly and annual data for output and unemployment equations are very compatible. But for the price equation, the two sets of data imply different results. Undoubtedly, the output and unemployment equations show that the regression coefficients between unanticipated money supply growth ( $UM_t$  and  $UM_{t-1}$ ) and the real variables of the model are statistically significant. However, the anticipated rate of growth of the money supply, other things remaining the same, has no effect on unemployment and output.

#### Neutrality of Anticipated Monetary Policies

The implication of neutrality (Lucas 1973, and Sargent and Wallace 1975) is that if aggregate demand policies are anticipated, then there will be no change in the real variables, such as real output and unemployment.

To investigate the neutrality of anticipated monetary policies, Mishkin used a nonlinear joint estimation of real output, the unemployment rate, and money supply growth (see Mishkin 1982).<sup>1</sup> If the assumptions

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<sup>1</sup>Mishkin's joint estimated model is based on  $Y_{jt} = Y_{jt}^* + \sum_{i=0}^N \beta_i (M_{t-i}^e - M_{t-i}^e)$  +  $\sum_{i=0}^N \delta_i M_{t-i}^e + \varepsilon_t$

where

$Y_{jt}$  = real output or the unemployment rate;  $Y_{jt}^*$  = natural unemployment rate;  $\delta_i, \beta_i$  = regression coefficients;  $M_t$  = the money supply growth;  $M_t^e$  = the expected money growth conditional upon all information in (t-1);  $\varepsilon_t$  = the error term.

of neutrality and rationality are rejected in a nonlinear joint estimation, then this is either due to irrational expectations, or because the anticipated policies have real effects (see Tylor 1975 and Fischer 1977). Alternatively, if the null hypothesis of macroeconomic rational expectations with its neutrality implication is rejected, then this in turn might be due to long lags existing in the output and unemployment equations.

The money supply growth used by Mishkin (1982) is derived by regressing the  $M_1$  - money supply growth on its four-quarter lagged values, as well as the three-month Treasury bill rate and the high employment budget surplus.<sup>1</sup> The money supply growth equation has excluded government expenditures and the unemployment rate, and unlike Barro and Rush who employed second-degree autocorrelation, Mishkin used a fourth-order auto-regression procedure.

The output equation in Mishkin's study, does not specify any lag length on  $(UM_t)$ , therefore one can go back as far as to when the lagged coefficients were no longer significant. This would suggest relatively short lags of 7 quarters. In the study completed by Gordon (1979), it was noted that the lags are much longer than what was suggested before, i.e., 20-quarters. In a joint estimation of rationality and neutrality with 7-quarter lags, these two propositions cannot be rejected. Nevertheless, when 20-quarter lags are employed, both of these two assumptions are re-

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<sup>1</sup>Among many other variables included in the money supply growth, these three were chosen by the multivariate Granger procedure.

jected, and comparably, the neutrality is rejected more significantly than the rationality proposition.

According to Mishkin, empirical results more favorable to the neutrality idea which underlie macroeconomic rational expectations are obtained in the context of misspecified models i.e., some explanatory variables are missing (for example, Barro and Rush 1980). For if data on anticipated money supply growth are included, then the regression coefficients and the t-statistics become more significant. The regression results of the output equation over the sample period (1959-1976) do not reject the neutrality assumptions. In the unemployment equation, the unanticipated money supply growth coefficients, at the five percent significance level, are significantly greater than zero. Neutrality of anticipated money supply growth is rejected, because of the significance of some of its coefficients, especially the last two lagged coefficients. This might mean that the inclusion of longer lags on anticipated and unanticipated money supply growth rates are essential for the rejection of neutrality and rationality propositions.

After the 20-quarter lags have been included in the jointly estimated model, the regression coefficients on the anticipated money supply growth rate are not only significant, but also have larger t-statistics than the unanticipated money supply growth rate coefficients. This result is contrary to the conclusion derived by other tests of macroeconomic rational expectations - most notably the study completed by Barro and Rush in 1980. Therefore, the anticipated money supply is not neutral, and its importance

cannot be disregarded (for details of the maximum likelihood ratio test supporting this idea, see Mishkin 1982, pages 34-39).

#### Interest Rates and Monetary Policies

One of the most interesting issues in economics is the relationship between the rate of growth of the money supply ( $g_M$ ) and interest rates ( $R$ ). For a long time, the Keynesian interest rate theory held that while other things remained equal, an increase in the quantity of money would decrease short- and medium-term interest rates. The inverse association between ( $g_M$ ) and ( $R$ ) directly followed from the law of supply, in which an increase in money supply decreased the price of borrowing money ( $R$ ).

A lower interest rate has a direct impact on investment, and also indirectly affects capital valuation, which results in further expansion of investment and consumable goods (see Modigliani, 1974). Correspondingly, a decline in the interest rate is essential for the transmission effects of monetary policies from the money market to the real sector of the economy. Most importantly, the psychology of a lower interest rate for the market participants is most appealing, especially when the monetary authority decides to increase the money supply through different monetary channels.

Keynesian liquidity models downplay the dynamic impact of a change in the money supply growth rate. According to Friedman (1968) and (1969 as cited by Mishkin (1983)), the liquidity theory ignores the income and price expectational effects. Suppose that the Federal Reserve unexpectedly increases the money supply growth rate. Consumers find themselves with

more cash than they had expected, therefore they spend extra money on different goods and services. One item that most commonly is purchased by the consumers are bonds of different maturities. A greater demand for bonds, in turn increases the price of bonds, which implies a decrease in bond yields.

The downward pressure on  $(R)$  is the short-run aspect of an unanticipated expansionary monetary policy. However, in the long-run, more money in the economy means a higher expected rate of inflation  $(\pi_t^e)$ . In essence, the relationship between  $(R_t)$  and  $(\pi_t^e)$  is through a Fisherian-type hypothesis. As Fisher (1930) mentioned, the nominal interest rate  $(R_t)$  is composed of the expected real interest rate  $E(r_t)$  and  $(\pi_t^e)$ . Furthermore, if it is naively believed that in the long-run,  $E(r_t)$  remains constant, then there is a one-to-one correspondence between  $(R_t)$  and  $(\pi_t^e)$ . Hence, an increase in  $(UM_t)$  ultimately means higher expected inflation rates, and a higher  $(\pi_t^e)$  implies a higher  $(R_t)$ , and vice versa.

The message given by Friedman and other monetary economists is that the liquidity effects of an increase in  $(UM_t)$ , which are indicated by a decrease in  $(R_t)$ , are very short-run. If the time period is long enough, then the liquidity effect is dominated by the price expectational effects, so that  $(g_M)$  and  $(R_t)$  are positively correlated.

Empirical work completed by Gibson (1970) and Cagan (1972) which is concerned with the relationship between  $(g_M)$  and  $(R_t)$ , concluded that the inflationary expectations "proceed slowly" over time. Thus, they placed more emphasis on the price level effects than on the price expectational effects. Nevertheless, Mishkin (1982) has shown that the mechanism for

forming price expectations is rather short, simply because under the assumption of rational expectations, the economic agent readjusts his/her expectations quickly. As such, "price anticipation effects" should be given more weight, when the impact of an increase in  $(g_M)$  on interest rates is considered.

Studies in line with Keynesian liquidity effects are faced with one major problem, because rational expectations (or equivalently financial market efficiency) have not been incorporated into the model. The evidence for the importance of efficient markets is so powerful, that in its absence the statistical results become doubtful (for example, see Fama 1970 and Mishkin 1978).

To account for the market efficiency, there are two different approaches. One alternative is to regress the first difference of interest rates on the changes of the past values of the money supply (see Gibson and Kaufman, 1968). This line of research diminishes the possibility of multicollinearity among explanatory variables. But due to reduced functional forms, the actual structure of the model cannot be specified, which may cause weak statistical results for such models. However, the results reported by these researchers do not confirm the inverse relationship between money supply growth and interest rates. The second alternative is to use anticipated and unanticipated money supply growth rates in the model. The latter approach has been used by Barro, Barro-Rush, and Mishkin, as well as in the present study. The decompositional money supply growth model is a better alternative, because in the context of



rational expectations, it specifies the exact structure of the model, and allows for a more powerful test of Keynesian propositions.

So far, the effects of an unexpected money supply growth on  $(R_t)$  in efficient bond markets have been discussed. The theory of the efficient market is also applicable if the money supply growth rate is anticipated by the general public. An increase in  $(AM_t)$  will increase  $(\pi_t^e)$  immediately, and from Fisher's hypothesis the increase in  $(AM_t)$  and  $(R_t)$  is positively correlated. This in turn implies that the expected real interest rate remains constant, which is directly due to the public's rational expectations, i.e., an increase in  $(AM_t)$  changes (increases) nominal variables  $(R_t)$ , but leaves the real variables  $E(r_t)$  unaffected. Consequently, if real variables such as real interest rates, unemployment, and real output are to be changed, then the money supply growth must come as a surprise to the public. In Barro's terminology, the anticipated growth rate of the money supply does not matter, because it leaves the real variables unchanged.

An important question in regards to the conclusion derived from above, can be raised - Can the monetary authority change its policies so frequently that the general public is fooled, and  $E(r_t)$  is affected, i.e., unanticipated monetary policy matters? The answer is twofold. First, rational expectations by definition mean the use of all available information, including that of the central bank's authority. So, whatever the change, the public should sooner or later catch onto it. Secondly, the central bank has some set (targeted) monetary policies, which if changed

frequently, are not different from "random policy making", i.e., not having a policy at all.

#### Concluding Remarks

Barro-Rush's unanticipated monetary policies on the basis of rational expectations are the beginning of a new era in economics. The implication of such policies for the monetary and fiscal authorities of a country is profound, in the sense that aggregate demand policies must come as a shock to the economic agent. Otherwise, none of the real variables such as real output, unemployment, relative prices, and the like can be affected.

Empirical studies completed in this area by Barro and Rush (1980) are based on a two-step regression procedure (as explained before). The two-step regression for models with shorter lags is an appropriate tool by which one can test the rationality and neutrality underlying ( $UM_t$ ) and ( $AM_t$ ) respectively. However, if long lags exist in the model, then according to Mishkin, a nonlinear joint estimation is needed.

Mishkin's model with shorter lags was very similar to that of Barro-Rush, with minor differences for sample periods, and in the choice of seasonally adjusted data. Hence, the difference revolved mainly around the longer lag length, because with 20-quarter lags included in the joint model, the statistical inferences were vastly different. As such, the rejection of neutrality and rationality underlying the macroeconomic rational expectations, as well as the unimportance of anticipated monetary policies is possible, when longer lags are present in the output and unemployment models.

Although the neutrality of systematic and deterministic policies are rejected by Mishkin's empirical results, there are at least three cautions in order; 1 - The longer lags in the money growth equation used as part of the nonlinear joint estimated model, might cause inefficiency due to the loss of degrees of freedom. The polynomial distributed lag function employed by Mishkin, is supposed to alleviate this problem; 2 - In Mishkin's model,  $Y_t^*$  (natural unemployment rate or output), is a measure of either a time trend, or the combined effects of variables such as the minimum wage and military conscription. However, these three variables have been included in Barro-Rush's model as separate explanatory variables; 3 - Mishkin's model is a reduced form, and still needs the justification of the unique solution when the estimated parameters are transformed from the reduced to the structured model.

The question of "does anticipated monetary policy matter" is still an open question. Although a series of papers by Barro and one by Barro-Rush cast doubt about the effectiveness of  $(AM_t)$ , all information about rational expectations, which is the main basis for the efficacy of  $(UM_t)$ , is not yet in. The only obvious conclusion derived from the whole controversy is that the growth rates of the money supply ( $AM_t$  and  $UM_t$ ) have two different directional effects (price expectational and liquidity) on the level of nominal interest rates. So, the question to be addressed is which one of these two effects is more significant in the overall picture of the economy. Most notably, under market efficiency, does an increase in  $(UM_t)$  have a dominant liquidity effect? The answer is what this research intends to resolve.

## SECTION III. INTEREST RATE MODELS

The backbone of the interest rate models is a demand function for money. Commonly in empirical money demand studies, the money supply (a proxy for the demand for money) is a dependent variable, while the nominal interest rate along with real income, and the price level appear as explanatory variables. Assuming that the level of the interest rate is the variable which changes when there is disequilibrium between the demand for and the supply of money, it appears sensible to regress the interest rate ( $R_t$ ) on the rate of growth of the money supply ( $g_M$ ), as opposed to the other way around.

The efficient market interest rate models produce useful results, because the direction of causation is from ( $g_M$ ) to ( $R_t$ ), especially if ( $g_M$ ) is decomposed between anticipated and unanticipated rates of growth of the money supply ( $AM_t$  and  $UM_t$ ). As was mentioned by Mishkin (1983), the interest rate models with the decomposed money supply growth produces far superior empirical testing of Keynesian liquidity effects to the models without market efficiency constraints.

Furthermore, since inflationary expectations are assumed to be fully incorporated into the nominal interest rate in efficient asset markets, it follows that if  $R_{t-1}$  is set at time ( $t$ ), then in assessing the expected rate of inflation ( $\pi_t^e$ ), other information (past inflation rates) is redundant (see Fama 1977). Under the efficient markets hypotheses, Fisher's formula can be amended to

$$[\pi_t^e | R_t, \phi_{t-1}] = -E(r_t | \phi_{t-1}) + R_t \quad (3-1)$$

where (E) stands for expectations and  $\phi_{t-1}$  is an informational set containing relevant information about  $(\pi_t^e)$ . The efficient market proposition implies that the market sets the price of bonds in such a manner that the expected real interest rate remains constant, i.e.,

$$E(r_t | \phi_{t-1}) = E(\bar{r}_t). \quad (3-2)$$

Although equation (3-2) is debatable, for market efficiency purposes, it is an appropriate approximation. It is also true that  $\phi_{t-1}$  includes a broader range of related information about  $(\pi_t^e)$ , but when  $(R_t)$  is set in  $(t-1)$ ,  $\phi_{t-1}$  does not supply new information in regards to  $(\pi_t^e)$ .

The interest rate models are designed to empirically test the degree to which short- and long-term interest rates are affected by the current and lagged values of  $(AM_{t-1})$  and  $(UM_{t-1})$ , i.e., two different directional effects. Additional lagged values were not included, because longer lags were not statistically significant. Therefore, the basic model is

$$R_t = F(AM_t, UM_t, AM_{t-1}, UM_{t-1}) + U_t \quad (3-3)$$

where (F) stands for a function of, the explanatory variables are measured as rates of growth, and  $(U_t)$  is a serially correlated disturbance term, i.e.,

$$U_t = \rho U_{t-1} + \varepsilon_t \quad (3-4)$$

where  $(\rho)$  is the autocorrelation coefficient and  $(\varepsilon_t)$  is a white noise error term. For  $(\varepsilon_t)$ , all classical normal error specificants are fulfilled. Hence,

$E(\varepsilon_t) = 0$ , i.e.,  $(\varepsilon_t)$  is a random variable

$E(\varepsilon_t, \varepsilon_{t-1}) = 0$ , i.e.,  $(\varepsilon_t)$  is not autocorrelated

$E(\varepsilon_t | AM_t, AM_{t-1}, UM_t, UM_{t-1}) = 0$ , i.e.,  $(\varepsilon_t)$  is uncorrelated with the regressors

$E(\varepsilon_t)^2 = \delta_{\varepsilon_t}^2$ , i.e.,  $(\varepsilon_t)$  is homoscedastic. Substituting equation (3-4)

into equation (3-3) gives

$$R_t = F(AM_t, UM_t, AM_{t-1}, UM_{t-1}) + \rho U_{t-1} + \varepsilon_t. \quad (3-5)$$

On the basis of model (3-5), the three different versions which will be used in empirical testing of the interest rate models are as follows:

$$R_t = \alpha_0 + \alpha_1 AM_t + \alpha_2 AM_{t-1} + \rho_1 U_{t-1} + \varepsilon_t \quad (3-6)$$

$$R_t = \beta_0 + \beta_1 UM_t + \beta_2 UM_{t-1} + \rho_2 W_{t-1} + \varepsilon_t' \quad (3-7)$$

$$R_t = \gamma_0 + \gamma_1 AM_t + \gamma_2 AM_{t-1} + \gamma_3 UM_t + \gamma_4 UM_{t-1} + \rho_3 Z_{t-1} + \varepsilon_t'' \quad (3-8)$$

where;

$R_t$  = interest rates

$AM_t, AM_{t-1}$  = anticipated rate of growth of the money supply, contemporaneously and one period lagged

$UM_t, UM_{t-1}$  = unanticipated rate of growth of the money supply, contemporaneously and one period lagged

$\alpha$ 's,  $\beta$ 's,  $\gamma$ 's = regression coefficients

$\rho_1, \rho_2, \rho_3$  = autoregressive correlation coefficients

$U_t, W_t, Z_t$  = serially correlated disturbance terms

$\varepsilon_t, \varepsilon_t', \varepsilon_t''$  = disturbance terms, not serially correlated with expected values of zero, and normal distributions.

For  $(R_t)$ , four different interest rates measured in annual percentage points have been used: 1 - Short-run Treasury bill rates (TBR) which are

the discount rates on new issues of 91-day Treasury bills. The data source is the Board of Governors of the Federal Reserve System. 2 - Commercial paper rates (CPR). The data are for the prime paper (4-6 months). The primary source is "Business Statistics 1977" for the period 1950-1976. The "Survey of Current Business 1978" has been used for the remaining quarters. 3 - Yields on long-term Treasury bond rates (LTR), "which are a measure of the average yield on fully taxable long-term U.S. Treasury bonds. Bond yields are computed by the U.S. Department of the Treasury, based on reported prices by the Federal Reserve Bank of New York. 4 - Yields on new issues of the high-grade (Aaa) corporate bond rate (LCR). Data has been computed by the Citibank (formerly the First National City Bank of New York) from 1949-1959 and the U.S. Department of the Treasury 1959-1977." The percentage interest rates data for TBR, LTR, and LCR are drawn from the Handbook of Cyclical Indicators - A Supplement to Business Conditions Digest - BCD (1977) and BCD (1978). Quarterly and yearly data on  $(UM_t)$  and  $(AM_t)$  have been taken from Barro-Rush's data set (1980).

The sample period for quarterly data covers 1950:I - 1978:I and for yearly data 1950-1977. Due to the problem of serially correlated disturbance terms, especially with quarterly data, all regression coefficients were corrected for serial correlation by employing the Cochrane-Orcutt corrective-procedure with a convergence level of 0.001.<sup>1</sup>

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<sup>1</sup>Since Barro and Rush estimated the money supply ( $M_1$ ) growth equation by using the Ordinary Least Squares (OLS) method, models 3-6, 3-7 and 3-8 were estimated using (OLS). For all the models, the Durbin-Watson was extremely small, indicating the existence of autocorrelation in the residuals.

If the  $(AM_t)$  series corresponds to what the public expects, then an increase in  $(AM_t)$  would increase the expected rate of inflation, and one would expect positive regression coefficients between  $R_t$  and the growth rate change of the anticipated portion of the money supply. Furthermore, in terms of Barro's definition of the unanticipated money supply growth, an increase in  $(UM_t)$  creates liquidity. More liquidity in turn puts a downward pressure on the interest rate. Hence, one would anticipate that the regression coefficient of  $(UM_t)$  would be negative.



## SECTION IV. EMPIRICAL RESULTS

## Quarterly Data

Model (3-6) has been tested empirically with the four different measures of interest rates used for  $R_t$ , and the results are reported in the following table. For all the models  $\rho$  is before, but  $R^2$  is after, the Cochrane-Orcutt procedure was applied.

Table 4-1. Regression results of  $R_t$  on  $AM_t$  and  $AM_{t-1}$ 

| $R_t$ | Intercept                    | $AM_t$                        | $AM_{t-1}$                    | $R^2$ | $\rho$ |
|-------|------------------------------|-------------------------------|-------------------------------|-------|--------|
| TBR   | 3.28<br>(3.39)* <sup>b</sup> | 40.95 <sup>a</sup><br>(2.56)* | 10.92 <sup>a</sup><br>(0.685) | 0.92  | 0.864  |
| CPR   | 3.93<br>(3.91)*              | 38.93<br>(1.87)               | 6.75<br>(0.326)               | 0.90  | 0.863  |
| LTR   | 4.68<br>(3.11)*              | 13.55<br>(2.31)*              | 1.24<br>(0.212)               | 0.98  | 0.869  |
| LCR   | 5.37<br>(3.25)*              | 24.06<br>(2.46)*              | 0.010<br>(0.001)              | 0.97  | 0.882  |

<sup>a</sup>Regression coefficients.

<sup>b</sup>Significant at the five percent significance level.

The numbers in parentheses are t-ratios, and the one indicated by an (\*) is larger than the tabled value of the t-statistic at the five percent significance level. The dependent variable ( $R_t$ ) is an annual interest rate measured in percentage points. The independent variables ( $AM_t$  and  $UM_t$ ) are

quarterly rates of growth of money supply, not measured in percentage points, and these symbols remain the same throughout.

According to the regression coefficients, interest rates are positively related to  $AM_t$ . However, the coefficient of  $AM_t$ , in the CPR regression, is not statistically significantly different from zero. The lagged values of  $AM_{t-1}$  do not reveal any significant relationship with the measures of interest rate.

Model (3-7), which is concerned with the sensitivity of  $R_t$  in respect to unanticipated money supply growth, shows the following regression results over the sample period.

Table 4-2. Regression results of  $R_t$  on  $UM_t$  and  $UM_{t-1}$

| $R_t$ | Intercept       | $UM_t$            | $UM_{t-1}$      | $R^2$ | $\rho$ |
|-------|-----------------|-------------------|-----------------|-------|--------|
| TBR   | 3.83<br>(3.51)* | -27.90<br>(3.09)* | 1.19<br>(0.132) | 0.92  | 0.961  |
| CPR   | 4.40<br>(4.12)* | -34.54<br>(2.96)* | 1.32<br>(0.113) | 0.90  | 0.947  |
| LTR   | 4.85<br>(3.14)* | - 5.61<br>(1.65)  | 2.69<br>(0.794) | 0.98  | 0.999  |
| LCR   | 5.64<br>(3.23)* | - 8.82<br>(1.55)  | 5.31<br>(0.934) | 0.97  | 0.988  |

Although only two interest rates TBR and CPR are inversely and significantly related to  $UM_t$ , the general inverse correlation between interest rates and unanticipated money supply growth is obvious. At least for short-run interest rates,  $UM_t$  creates a liquidity-effect, which in turn

puts a downward pressure on  $R_t$ . But the liquidity impact of  $UM_t$  on the long-run interest rate, is not statistically significant.

To investigate the effect of  $AM_t$  and  $UM_t$  on  $R_t$  simultaneously, model (3-8) has been empirically tested, and the results are as follows.

Table 4-3. Regression results of  $R_t$  on  $AM_t$ , and  $UM_t$ , and their one period lag

| $R_t$ | Intercept       | $AM_t$           | $AM_{t-1}$        | $UM_t$            | $UM_{t-1}$       | $R^2$ | $\rho$ |
|-------|-----------------|------------------|-------------------|-------------------|------------------|-------|--------|
| TBR   | 3.35<br>(3.17)* | 74.77<br>(1.99)* | -26.70<br>(1.20)  | -27.06<br>(2.73)* | -40.71<br>(1.76) | 0.93  | 0.868  |
| CPR   | 4.10<br>(3.66)* | 68.62<br>(1.41)  | -37.54<br>(1.30)  | -36.56<br>(2.83)* | -44.36<br>(1.48) | 0.91  | 0.864  |
| LTR   | 4.68<br>(3.08)* | 24.08<br>(1.69)  | - 7.48<br>(0.898) | -5.10<br>(1.36)   | -10.41<br>(1.19) | 0.98  | 0.806  |
| LCR   | 5.32<br>(3.16)* | 49.04<br>(2.07)* | -17.43<br>(1.25)  | -8.25<br>(1.32)   | -22.15<br>(1.52) | 0.97  | 0.838  |

Two different directional impacts of  $AM_t$  and  $UM_t$  are inferred from Table 4-3. That is, an anticipated rate of growth of the money supply creates price expectational effects on different short- and long-run measures of interest rates. TBR and LCR are significantly related to  $AM_t$ , although the t-statistic for TBR is very close to the critical value of the t-distribution ( $t=1.96$ ) at the 5 percent level.  $UM_t$  does not have a significant relationship with LTR and LCR, whereas the two short-run interest rates (TBR and CPR) display significant liquidity effects, due to an unanticipated rate of growth of the money supply. The one-quarter lagged values of  $AM_t$  and  $UM_t$ , are not statistically significantly correlated with short- and long-run interest rates.

On the basis of the models explained in the previous three tables, the impact of an annual one percentage point increase in the money supply growth rate (percent  $AM_t$  and percent  $UM_t$ ) on the interest rates ( $R_t$ ) is reported in the following table.

Table 4-4. Percent change in interest rates due to a one percent rate of growth of the money supply per year.

| Models in table | $R_t$ | $AM_t^*$ | $UM_t^*$ | % $AM_t$ | % $UM_t$ |
|-----------------|-------|----------|----------|----------|----------|
| (4-1)           | TBR   | 40.95    |          | 0.102    |          |
|                 | LTR   | 13.55    |          | 0.034    |          |
|                 | LCR   | 24.06    |          | 0.060    |          |
| (4-2)           | TBR   |          | -27.90   |          | -0.069   |
|                 | CPR   |          | -34.54   |          | -0.086   |
| (4-3)           | TBR   | 74.77    | -27.06   | 0.187    | -0.067   |
|                 | CPR   |          | -36.56   |          | -0.091   |
|                 | LCR   | 49.04    |          | 0.122    |          |

\* Significant quarterly money supply growth rate coefficients from Tables 4-1, 4-2, and 4-3.

The price expectational effects of a one percent increase in ( $AM_t$ ) and ( $R_t$ ), although small in absolute value, are larger than the liquidity effect of the same rate of increase in ( $UM_t$ ) on models reported in Table 4-3. Nevertheless, models in Table 4-2 show a more significant impact of percent  $UM_t$  on short-term interest rates, whereas the primary effects of percent  $AM_t$  in Table 4-1, are on the long-term rates and TBR.

## Yearly Data

As in Barro and Rushs' paper, we have tested the sensitivity of interest rates to  $AM_t$  and  $UM_t$ , by using annual data.<sup>1</sup> Using annual data decreases the problem of autocorrelation. However, the ordinary least squares estimation of regression coefficients with yearly data, revealed a relatively high serial correlation in the residuals. Thus, all the following models are corrected for serial correlation, by employing the Cochrane-Orcutt corrective procedure. Model (3-6) has been tested with yearly data, and the results are reported in the following table.

Table 4-5. Regression results of  $R_t$  on  $AM_t$ , and  $AM_{t-1}$ 

| $R_t$ | Intercept       | $AM_t$           | $AM_{t-1}$       | $R^2$ | $\rho$ |
|-------|-----------------|------------------|------------------|-------|--------|
| TBR   | 3.55<br>(3.97)* | 4.93<br>(0.473)  | 13.43<br>(1.28)  | 0.68  | 0.627  |
| CPR   | 4.24<br>(4.43)* | 1.54<br>(0.119)  | 15.26<br>(1.17)  | 0.62  | 0.573  |
| LTR   | 4.76<br>(3.35)* | -1.72<br>(0.464) | 1.73<br>(0.466)  | 0.92  | 0.863  |
| LCR   | 5.71<br>(3.26)* | -4.56<br>(0.718) | 0.441<br>(0.069) | 0.90  | 0.808  |

Excluding the intercept, none of the regression coefficients are statistically significant. Considering the high  $R^2$  for long-term interest

<sup>1</sup>Interest rates are annual rates measured in percent points, but ( $AM_t$ ) and ( $UM_t$ ) are yearly rates of growth of money supply.

rates, the insignificant results might have been due to the multicollinearity between  $(AM_t)$  and its lagged value. For the (LTR) model,  $(\rho)$  has not changed despite the switch from quarterly to yearly observations. This in turn implies that the (LTR) model is either misspecified, or has excluded important explanatory variables. The price expectational effects of  $(AM_t)$  with quarterly data were significant for TBR, LTR, and LCR. But the annual data infer vastly different results.

Model (3-7) has been reestimated by utilizing yearly data, and the results are:

Table 4-6. Regression results of  $R_t$  on  $UM_t$ , and  $UM_{t-1}$

| $R_t$ | Intercept       | $UM_t$             | $UM_{t-1}$        | $R^2$ | $\rho$ |
|-------|-----------------|--------------------|-------------------|-------|--------|
| TBR   | 3.82<br>(3.33)* | -141.08<br>(2.55)* | 42.06<br>(0.762)  | 0.76  | 0.789  |
| CFR   | 4.51<br>(3.86)* | -161.24<br>(2.19)* | 11.38<br>(0.155)  | 0.67  | 0.736  |
| LTR   | 4.92<br>(3.04)* | - 52.50<br>(2.50)* | -14.80<br>(0.706) | 0.94  | 0.954  |
| LCR   | 5.96<br>(2.87)* | - 97.35<br>(2.83)* | -60.59<br>(1.76)  | 0.92  | 0.922  |

The unanticipated growth rate of money supply is inversely and significantly correlated to short- and long-run interest rates. In other words,  $UM_t$  creates a liquidity effect on the bond market with annual observations on  $(R_t)$  and  $UM_t$ , as was true with the quarterly data. However, with quarterly data, the regression coefficients of LTR and LCR are not statistically significant.

Model (3-8), which incorporates  $AM_t$  and  $UM_t$  at the same time, was tested with yearly data, and the results are as follows:

Table 4-7. Regression results of  $R_t$  on  $AM_t$  and  $UM_t$ , and their one period lag

| $R_t$ | Intercept       | $AM_t$          | $AM_{t-1}$       | $UM_t$             | $UM_{t-1}$        | $R^2$ | $\rho$ |
|-------|-----------------|-----------------|------------------|--------------------|-------------------|-------|--------|
| TBR   | 3.75<br>(3.15)* | 4.78<br>(0.436) | 2.94<br>(0.278)  | -148.51<br>(2.32)* | 24.99<br>(0.358)  | 0.76  | 0.651  |
| CPR   | 4.39<br>(3.67)* | 3.99<br>(0.273) | 6.72<br>(0.477)  | -157.57<br>(1.85)  | -7.49<br>(0.080)  | 0.68  | 0.593  |
| LTR   | 4.91<br>(2.91)* | 0.65<br>(0.155) | 0.469<br>(0.117) | -53.40<br>(2.19)*  | -17.24<br>(0.640) | 0.93  | 0.857  |
| LCR   | 5.94<br>(2.73)* | 2.95<br>(0.433) | 0.909<br>(0.138) | -104.14<br>(2.62)* | -70.24<br>(1.61)  | 0.92  | 0.817  |

Comparing Table 4-3 with Table 4-7 reveals the same sign pattern, except for  $AM_{t-1}$  which is negative with quarterly data, but was not significant. TBR and LCR, which were significantly correlated to anticipated change in the money supply when quarterly data were used, are not statistically significant. The significance level for unanticipated money supply growth has increased in Table 4-7, in which yearly data are utilized. With quarterly data, the liquidity impact of  $UM_t$  was a dominant factor for short-run interest rates. Whereas with yearly data, TBR and the two measures of long-run interest rates are inversely and significantly correlated with  $UM_t$ .

As with quarterly observations, the following table shows by how much interest rates rise or drop if money growth is increased by one percent per year, and whether this is anticipated or unanticipated.

Table 4-8. Percent increase in interest rates due to a one percent increase in money supply per year

| Models in Table | $R_t$ | $AM_t^*$ | $UM_t^*$ | % $AM_t$ | % $UM_t$ |
|-----------------|-------|----------|----------|----------|----------|
| (4-6)           | TBR   |          | -141.08  |          | -1.4108  |
|                 | CPR   |          | -161.24  |          | -1.6124  |
|                 | LTR   |          | - 52.50  |          | -0.5250  |
|                 | LCR   |          | - 97.35  |          | -0.9735  |
| (4-7)           | TBR   |          | -148.51  |          | -1.4851  |
|                 | LTR   |          | - 53.40  |          | -0.5340  |
|                 | LCR   |          | -104.14  |          | -1.0414  |

\*Significant annually money supply growth coefficients from Tables 4-5, 4-6, and 4-7.

Clearly, with yearly data the liquidity effect of a one percent annual increase in ( $UM_t$ ) has a significant impact on interest rates. The same rate of growth in ( $AM_t$ ) does not appear to have any significant relationship with ( $R_t$ ). The two different statistical results, when the time interval is quarterly and annually, are due to a change in the residual autocorrelations, and the sample size. That is, yearly observations cover some of the autocorrelation in the error-term, but with a smaller sample size some degrees of freedom are lost, which in turn creates inefficient statistical inferences.



## SECTION V. SUMMARY AND CONCLUSIONS

The primary purpose behind testing the sensitivity of different measures of interest rates, with respect to both anticipated ( $AM_t$ ) and unanticipated ( $UM_t$ ) money supply growth, was to answer the following questions.

1 - Do  $AM_t$  and  $UM_t$  have two different directional effects? According to the empirical results, this is so in that  $AM_t$  is positively, and in some cases significantly, correlated with short- and long-run interest rates. Also,  $UM_t$  is inversely and significantly related to interest rates.

2 - Are the two directional impacts on interest rates in agreement with the notion of efficient markets? The statistical results tend to supply a yes to this question. An increase in  $AM_t$  is associated with a consequent increase in the nominal interest rate. Whereas, an increase in  $UM_t$  creates a liquidity effect which puts downward pressure on the nominal interest rate.

3 - Does anticipated money supply growth matter? Interest rate models which were tested empirically, indicate that this is so with the quarterly data, but not so with the annual time series data. Recall that all the models were adjusted for serial correlation and the possible heteroscedasticity problem. Also, because of a two-step regression method used in this study, the money supply growth ( $AM_t$  and  $UM_t$ ) is exogenous, i.e., it depends on past events - the possibility of any correlation between the explanatory variables ( $AM_t$  and  $UM_t$ ) and the error term is minimal. So, the t-test results cannot be biased.

4 - Do the dominant price level expectational effects of  $AM_t$  and the liquidity effects of  $UM_t$  depend upon a particular time interval aggregation? There is a yes to this question. For a one percent increase in  $AM_t$ , quarterly data imply a more significant increase in interest rates (price expectational effects). Whereas, for the same increase in  $UM_t$ , yearly data show a significant drop in interest rates (liquidity effects).

It is a rather difficult task to assess which of the two time aggregations is better. Generally speaking, when the time span is lengthened, there are more possibilities for statistical problems, due to the reduction of the sample size, and the coverage of the residual autocorrelation. As was witnessed by the autocorrelation coefficient ( $\rho$ ), quarterly data (except for LTR in Table 4-7) detect more serious serial correlation than does yearly data. It is worth mentioning that the LTR model in Table 4-7 is faced with impure autocorrelation, as evidenced by a yearly ( $\rho$ ) which is larger than the quarterly ( $\rho$ ). Thus, LTR in Table 4-7 might be misspecified, or it might have excluded important explanatory variables.

In summary, the interest rate models used in this study are subject to the common problems associated with the two-step regression procedure, employed by Barro-Rush. That is, the underlying money supply growth is in a reduced form, and its residual  $UM_t$  is orthogonal to its explanatory variables. However, the decompositional money supply growth model is not "the end of the rational expectations revolution in macroeconomics", but rather is "the end of the beginning".

## REFERENCES

- Barro, Robert J. "Rational Expectations and the Role of Monetary Policy." Journal of Monetary Economics 2 (1976): 1-33.
- Barro, Robert J. "Unanticipated Money, Output, and the Price Level in the United States." Journal of Political Economy 86 (August) 1978: 549-80.
- Barro, Robert J. "Unanticipated Money, Output, and the Price Level in the United States." In R. Barro (ed.) Money, Expectations, and Business Cycles. New York: Academic Press, 1981.
- Barro, Robert J. Macroeconomics. New York: John Wiley and Sons, Inc., 1983.
- Barro, Robert J. and Mark Rush. "Unanticipated Money and Economic Activity." In Stanley Fischer (ed.) Rational Expectations and Economic Policy. Chicago, University of Chicago Press, 1980.
- Begg, David K.H. "The Rational Expectations Revolution in Macroeconomics - Theories and Evidence." Baltimore, Maryland: The Johns Hopkins University Press, 1982.
- Cagan, Phillip. "The Channels of Monetary Effects on Interest Rates." New York: National Bureau of Economic Research, 1972.
- Fama, Eugene F. "Efficient Capital Markets: A Review of Theory and Empirical Work." Journal of Finance 25 (1970): 283-417.
- Fama, Eugene F. "Interest Rates and Inflation: The Message in the Entrails." American Economic Review 67, No. 3 (June, 1977): 487-496.
- Fischer, Stanley. "Long Term Contracts, Rational Expectations, and The Optimal Money Supply Rule." Journal of Political Economy 85, No. 1, (February, 1977): 191-206.
- Fisher, Irving. The Theory of Interest. New York: The Macmillan Company, 1930.
- Friedman, Milton. "The Role of Monetary Policy." American Economic Review 58 (1968): 1-17.
- Gibson, William E. "Interest Rates and Monetary Policy." Journal of Political Economy 78 (1970): 431-455.

- Gibson, W. E., and G. G. Kaufman. "The Sensitivity of Interest Rates to Changes in Money and Income." Journal of Political Economy 76 (1968): 472-478.
- Gordon, Robert J. "New Evidence That Fully Anticipated Monetary Changes Influence Real Output After All." Discussion paper No. 369. Northwestern University, Center for Mathematical Studies in Economics and Management Science, Evanston, Illinois, 1979.
- Lucas, Robert E., Jr. "Expectations and Neutrality of Money." Journal of Economic Theory 4 (1972): 103-24.
- Lucas, Robert E., Jr. "Some International Evidence on Output-Inflation Tradeoffs." American Economic Review 63 (June, 1973): 324-34.
- Mishkin, Frederic S. "Efficient Markets Theory: Implications for Monetary Policy." Brookings Paper on Economic Activity 3 (1978): 707-752.
- Mishkin, Frederic S. "Does Anticipated Monetary Policy Matter? An Econometric Investigation." Journal of Political Economy 90 (February, 1982): 22-51.
- Mishkin, Frederic S. A Rational Expectations Approach to Macroeconomics Testing Policy Ineffectiveness and Efficient Market Models. Chicago: University of Chicago Press, 1983.
- Modigliani, Franco. "The Channels of Monetary Policy in FMP Econometric Model of the U.S." In G. Renton (ed.), Modelling the Economy. London: Heineman Educational Books, 1974.
- Sargent, Thomas J. "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment." Brookings Papers on Economic Activity 2 (1973): 429-72.
- Sargent, T. J. and N. Wallace. "Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule." Journal of Political Economy 83 (1975): 241-54.
- Taylor, John B. "Monetary Policy During the Transition to Rational Expectations." Journal of Political Economy 83 (1975): 1009-21.

PART TWO  
WITHER THE DARBY EFFECT

## SECTION I. INTRODUCTION

Irving Fisher (1930) stated that the expected rate of inflation is fully captured in the market rate of interest. Furthermore, if in the long-run the expected real rate of interest is constant, then there is a one-to-one relationship between an increase in the expected inflation rate ( $\pi^e$ ) and the nominal interest rate ( $i$ ).

There are several major problems involved with the Fisherian hypothesis. First, it is not obvious how inflation expectations are formed. Second, a constancy for the expected real interest rate ( $r_t^e$ ) is not plausible, because there are a number of factors affecting ( $r_t^e$ ), which have not been captured in Fisher's Simple formulation. Third and most importantly, as Darby (1975) and Feldstein (1976) almost simultaneously but independently argued, interest payments/receipts on a bond are tax-deductible and subject to income tax, respectively. However, Fisher's formula does not capture the tax effect, and thereby simplifies reality a great deal.

Econometrically, the Fisherian hypothesis implies that in regressions explaining the nominal interest rate, the coefficient of the expected rate of inflation would be unity. However, incorporating the income tax consideration (known as the Darby effect) requires the coefficient of ( $\pi^e$ ) to be greater than one. That is, ( $i$ ) must increase by more than the increase in ( $\pi^e$ ) in order to make up for both inflation losses, and taxes paid on interest received from a bond.

Over the past decade, the tests of the Darby effect have mainly been aimed at whether the regression coefficient of  $(\pi^e)$  is greater than unity. Examples of some of the authors whose work is directly related in this regard, include John Carlson (1979), Vito Tanzi (1980), Michael Melvin (1982), Milton Ezrati (1982), Lew Silver and James Fackler (1982), and Robert Ayanian (1983).

All the studies mentioned above, excluding Ayanian's, concluded that the coefficient of  $(\pi^e)$  is less than unity. Nevertheless, they did not claim that they had disproven the Darby effect. One researcher placed the blame on risk associated with the returns on capital. Others blamed incorrect expectations of inflation or the variability of  $(r_t^e)$  for the lack of empirical support for the Darby effect. Ayanian, by setting aside the data on  $(\pi^e)$  and the expected real interest rate, empirically tested (in the context of some simplifying assumptions) the same model advocated by Darby and Feldstein. He concluded that over his sample period (1952-1979) the coefficient of  $(\pi^e)$  was in fact greater than unity, thereby proving that the Darby effect did exist.

The main goal of this study is to reestimate Ayanian's model, by using monthly and quarterly data for the period 1952-1979. The point of departure from that of Ayanian's is to argue that his model is heavily affected by residual autocorrelation and thereby violates one of the most basic classical econometric assumptions, which is needed if the regression coefficient is to be BLUE (best linear unbiased estimates). Thus, Ayanian's model, with such a high residual autocorrelation, is not a good

test of the Darby effect. Then, Ayanian's model is extended from 1979 to 1983, to determine the coefficient of ( $\pi^e$ ), and it is extended to the entire period 1952-1983 using both monthly and quarterly data.

A review of the related literature appears in Section II, Section III is devoted to the empirical results, and the conclusion of the study is discussed in Section IV.



## SECTION II. A REVIEW OF RELATED LITERATURE

## Fisher Effects

Suppose that there is a one-year bond with a face value of \$1, and an annual rate of return (nominal interest rate) of  $i_t$ . Therefore, the nominal value of the bond at the end of the year is  $(1 + i_t)$ . If the general price level remains constant permanently, then  $(1 + i_t)$  is also the real value of the bond. However, inflation drives a wedge between the nominal and real value of an asset.

If the inflation rate is denoted by  $(\pi_t)$ , and the general price level at time  $(t)$  by  $P_t$ , then  $(\pi_t)$  can be referred to as;

$$\pi_t = \frac{P_{t+1} - P_t}{P_t} \quad (2-1)$$

or equivalently as;

$$P_{t+1} = (1 + \pi_t)P_t \quad (2-1)'$$

Equation (2-1)' indicates that between time  $(t)$  and  $(t+1)$ , the price level has grown by a  $(1 + \pi_t)$  factor.

In terms of the real interest rate  $(r_t)$ , the real value of the bond after one year is  $(1 + r_t)$ , which by definition can be also viewed as;

$$(1 + r_t) = (1 + i_t)/(1 + \pi_t) \quad (2-2)$$

$$\text{or } 1 + r_t + \pi_t + r_t\pi_t = 1 + i_t \quad (2-2)'$$

the term  $r_t\pi_t$ , known as the interaction effect, is very small and reasonably close to zero, unless the inflation rate is quite high. Thus, equation (2-2)' can be simplified to;

$$i_t = r_t + \pi_t. \quad (2-3)$$

Although  $i_t$  is observable in the bond market,  $r_t$  depends also on the actual rate of inflation ( $\pi$ ). It is assumed that the individual investors form their expectations rationally in regards to the future inflation rate. "Rationally" implies that one uses all available information and revises it in order to minimize the possibility of systematic errors. As such, equation (2-3) can be amended to;

$$i_t = r_t^e + \pi_t^e \quad (2-4)$$

where  $r_t^e$  is the expected real interest rate (see Barro 1983).

Equation (2-4) is the Simple Fisher formula in which Fisher's effects impose two related fundamental restrictions. One such restriction is that the expected real return on the bond, over a long-run time period, remains constant. The second, is that if  $(\pi_t^e)$  changes by  $\chi$  percent, then  $(i_t)$

changes by the same exact percentage point - such that  $\frac{\Delta i_t}{\Delta \pi_t^e} = 1$ .

#### Darby Effects

According to Darby (1975), the problem involved with the simple Fisherian hypothesis is two-fold. First of all, there is inconsistency involved between what the interest rate is in theory and what it is when tested empirically. Theoretically,  $(i_t)$  in Fisher's formula has been thought of as an average phenomenon, i.e., the expected value of the interest rate which prevails in all the asset markets, whereas, the empirical tests have used a very specific interest rate (such as the Treasury bill rate). Secondly, as was mentioned at the outset, there is no room for interest income taxes.

If interest income taxes are considered, then a rational market would equate the nominal after-tax interest rate to the expected after-tax real interest rate ( $r_t^e$ ) plus the expected rate of inflation. Assuming a proportional tax rate of ( $0 < T < 1$ ), the Darby formula can be written as;

$$(1 - T)i_t = r_t^e + \pi_t^e \quad (2-5)$$

$$\text{or } i_t = r_t^e / (1 - T) + \pi_t^e / (1 - T) \quad (2-5)'$$

Equation (2-5)' is the modified or corrected (for taxes) version of the Fisher hypothesis. The Darby effect in equation (2-5)' implies that  $\Delta i_t / \Delta \pi_t^e = 1 / (1 - T) > 1$ . Hence, a one percent increase in ( $\pi_t^e$ ) corresponds to a more than one percentage point increase in ( $i_t$ ), so that the lenders are compensated for both ( $\pi_t^e$ ) and the taxes that they must pay on interest income. For a progressive tax rate system, equation (2-5)' remains applicable if the marginal lenders' marginal tax rate remains constant.

The Darby formula can be elaborated on by the investigation of a change in ( $\pi_t^e$ ), and the subsequent impact on ( $i_t$ ). Also, unlike the Fisher formula in which ( $r_t^e$ ) is constant, here one can determine which factors have the potential for affecting ( $r_t^e$ ). The expected rate of inflation ( $\pi_t^e$ ) is most sensitive to a change in the rate of growth of the money supply, because in the long-run, inflation is purely a monetary phenomenon. In the short-run, although the growth rate of the money supply ( $g_M$ ) and ( $\pi_t$ ) are closely related, changes in the rate of growth of real income decrease the one-to-one relationship between ( $g_M$ ) and ( $\pi_t$ ), known as the price expectational effect.

A change in  $(g_M)$  will also create liquidity and income effects. The liquidity effects are associated with a drop in the nominal interest rate, which directly follows the Keynesian interest rate theory. On the other hand, the income effect is caused by the transmission mechanism, which is a channel through which the liquidity effect is transmitted from the monetary to the real sector of the economy. In essence, a decrease in the nominal interest rate stimulates investment, which through the multiplier effect increases income.

The three effects of an increase in  $(g_M)$  complicate the impact of  $(\pi_t^e)$  on the nominal interest rate. This is because  $(i_t)$  is partially a function of the substitutability between different assets, which usually occurs when returns of different assets are affected differently by the rate of growth of the money supply. To word the matter differently, an increase in  $(g_M)$  has both direct effects (price expectational, liquidity, and income effects), and indirect effects (substitution effects) on the nominal interest rate. Therefore, the link between  $(\pi_t^e)$  and  $(i_t)$  is much too complex to be captured via a simple equation such as (2-5)'.

In regards to the question of whether  $(r_t^e)$  remains constant, a number of factors should be considered, including the nature of the aggregate production function, the national saving function, and the status of the economy - all of which are of particular importance. As a highlight, assume a neoclassical growth model in which the population grows at rate  $(n')$ , and the per capita income  $(Y/n)$  is a function of the capital labor ratio  $(K/n)$ . Steady state equilibrium requires that there must be enough

capital to equip new laborers such that  $(K/n)$  remains constant. Furthermore, assume that the real per capita demand for money  $(M^d/Pn)$  where  $(P)$  is the price level, is positively related to  $(Y/n)$ , and inversely related to  $(\pi_t^e)$  and the return on physical capital.

The per capita saving  $(S/n)$  is a constant fraction  $(\bar{S})$  of  $(Y/n)$  minus per capita consumption, which itself is dependent on disposable income. In this model, disposable income is defined as the percentage rate of growth of the money supply  $g_M$  minus  $(\pi_t)$ , multiplied by  $(M^d/Pn)$ .<sup>1</sup> If money is outside money, then an increase in  $(g_M)$  increases  $(\pi_t)$  and decreases  $(M^d/Pn)$ . Therefore, consumption per capita decreases, while  $(S/n)$  increases, which implies that the per capita saving function shifts upwardly. Since the neoclassical model is always in equilibrium, i.e., planned saving is always equal to planned (actual) investment, then a higher  $(S/n)$  corresponds to a higher  $(K/n)$ . Hence, money is nonneutral and is capable of changing the real interest rate.

Another case of the nonneutrality, is when money is inside money, and real per capita money balances are considered as a factor of production. An increase in  $(g_M)$  increases the price level proportionately, as well as increasing  $(\pi_t^e)$ , which subsequently decreases  $(M^d/Pn)$ . The equilibrium position in the money market necessitates a decline in the supply of real money balances. As such, the production function and the corresponding

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<sup>1</sup> $\frac{S}{n} = \bar{S} \frac{Y}{n} - \bar{C}(g_M - \pi_t) \frac{M^d}{pn}$ , where  $\bar{C} = (1-\bar{S})$ , and the money market is in equilibrium, i.e.,  $\frac{M}{pn}$  equals the per capita supply of real balances.

saving function shift downward, which implies a lower  $(K/n)$  at the new steady state equilibrium (see Harris 1981).

#### Darby Effects under Uncertainty

The Darby effect treats the return on different assets as though they are equally safe, which means the formula is applicable in a world of perfect foresight. However, in a world of incomplete information in which uncertainty develops, risk consideration must be incorporated into the model (see Carlson 1979). This modification is important, because beyond tax considerations, there are more risks associated with the returns on physical capital than the returns on bonds. In the context of a one-good model, the Darby formula can be modified as (see Carlson, 1979 page 599);

$$i_t = \mu_1 F'(K) + \mu_2 \pi_t / (1-T); F'_K > 0, F''_K < 0 \quad (2-6)$$

where  $F'(K)$  is the marginal physical productivity of capital (expected real returns on capital), and  $\mu_1$  and  $\mu_2$  are the two variable fractions that are meant to capture uncertainty in the model.

The existence of these two risk factors, obviously decreases the correspondence between  $F'(K)$ ,  $\pi_t^e$ , and the nominal interest rate. For example, a value close to one for  $\mu_1$ , infers the same degree of safety on the returns of capital as it does for bonds. Whereas, if  $\mu_1$  falls below unity, then uncertainty corresponding to the returns on capital grows. Meanwhile,  $\mu_1$  reflects the degree to which the capital stock has been utilized, or it stands for the capacity effect of capital on  $(i_t)$ . That is, a rising  $\mu_1$  stands for full-utilization of capital, while a fall in  $\mu_1$  is a sign of idle capital.

Although interest payments are tax exempt and interest income is subject to tax, the appreciation value of capital goods is not fully taxable. In essence, while output produced by capital is taxed, the appreciation rate of capital, which must grow at the same rate as the commodity prices, is partially tax exempt. Hence,  $\mu_2$  in equation (2-6) is designed to capture the effect on  $(i_t)$  of a change in expected relative prices of capital and other goods.

In the presence of uncertainty regarding the returns on capital, the expected real interest rate cannot remain constant because the utilization rate of capital changes over time. Therefore, the capacity effect must be added to the model as a new explanatory variable. In the empirical test completed by Carlson (1979), the capacity and liquidity effects were added to the regression equation. Following Lahiri (1976), the extrapolative expected rate of inflation was thought to be subject to random disturbances. Thus, the simple ordinary least squares method (OLS) infers biased results, because the error term and the explanatory variables are correlated. Thus, by using a two-stage least squares method (2SLS), coupled with the time series processor which executes the Cochrane-Orcutt procedure in (2SLS), Carlson concluded that the Darby effects in the 1950s and between 1970-75 did not exist. However, in the 1960s the Darby effect was experienced, which could have been due to taking  $(\pi_t^e)$  into account in regards to the returns on capital. The capacity parameter, especially in the 1970s, shows a significant coefficient which indicates that  $(i_t)$  has been heavily affected by the capacity effect over

the sample period. The liquidity effect of an increase in the rate of growth of the money supply and the interest rate (the 4-6 month commercial paper rate) are also significant.

#### Darby Effects in a Fluctuating Economy

One of the building blocks of empirical testing of Fisher's and Darby's formulas is the way in which  $(\pi_t^e)$  is formed. Traditionally, this was done by making direct use of Joseph Livingston's survey data (for example, see Gibson, 1972). On the same grounds, Lahiri (1976) tried to form  $(\pi_t^e)$  on the basis of observed prices expectations, along with the past rates of inflation. To accomplish this goal, Lahiri used four different versions of  $(\pi_t^e)$ ; distributed lag, adaptive, extrapolative, and Frenkel's derived version. He also used (2SLS), by which in the first stage, he estimated four versions of the expected rate of inflation, then in the second stage substituted an estimated  $(\pi_t^e)$  into Fisher's hypothesis. Although short-run interest rates were used, all of the regression coefficients of  $(\pi_t^e)$  were less than one.

Lahiri's model infers wrong statistical results, because on one hand, there is no consistency in forming expectations and the interest rates used. On the other hand, his model is misspecified (see Tanzi, 1980). The resulting inconsistency is due to the fact that returns on 3-month Treasury bills are associated with a 6-month  $(\pi_t^e)$ . It is also due to the fact that a wrong specification, because the level of economic activity, which is an important determinant of the interest rate, has been left out.



The level of economic activity can affect the actual real interest rate either by affecting the expected rate of inflation, or by affecting the expected real interest rate. Referring to equation (2-4) as mentioned earlier, indicates that the expected inflation rate is subject to random error, which creates a discrepancy between ( $\pi_t^e$ ) and the actual rate of inflation ( $\pi_t^a$ ). From Fisher's hypothesis, it is easily deduced that;

$$r_t^e = i_t - \pi_t^e \quad (2-7)$$

If the stochastic factors do not exist, then  $\pi_t^e = \pi_t^a$  and the realized interest rate ( $r_t^r$ ) is derived from the following relationship;

$$r_t^r = i_t - \pi_t^a \quad (2-8)$$

subtracting (2-8) from (2-7) results in;

$$r_t^e - r_t^r = \pi_t^a - \pi_t^e = Z_t. \quad (2-9)$$

$Z_t$  is assumed to have an expected value of zero and no serial correlation with lagged  $Z_t$ . These two classical assumptions hold true as long as the level of economic activity is constant. During expansionary and contractionary periods,  $Z_t$  does not fulfill these two requirements. However, through the augmented Phillips curve hypothesis and Okun's law,  $Z_t$  can be linked to the index of economic activity<sup>1</sup> (see Dornbusch and Fischer, 1978). If income is above its full employment level, then ( $i_t$ ) rises more than  $\pi_t^e$ . The opposite is true during a recessionary period. Therefore,  $i_t$  and  $r_t^e$  are directly linked to the ups and downs of the level of economic activity.

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<sup>1</sup> $i_t = r_t^n + \theta(Y - \bar{Y})$ , where  $r_t^n$  is the natural real rate of interest,  $\theta$  is the coefficient of the index of economic activity,  $Y$  is actual, and  $\bar{Y}$  is the potential level of income (see Tanzi 1980, page 16).

The relationship between  $(r_t^e)$  and the index of economic activity has been tested in a few studies. Elliott (1977) found an insignificant relationship between  $(r_t^e)$  and the rate of real output. Fama (1977) concluded that there was a direct relationship between changes in  $(\pi_t^e)$  and  $(i_t)$ . Tanzi (1980) summarized that the inclusion of economic activity improved the regression coefficient of  $(\pi_t^e)$  and the goodness of fit.

Tanzi (1980) tested for the existence of the Darby effect by calculating an average tax rate of  $T = 0.32$ , over the sample period 1952-1975. In terms of the Darby equation, the coefficient of  $(\pi_t^e)$ , when taxes were present  $(1/1-T)$ , implied a coefficient of 1.47. This coefficient implied that lenders have been compensated for  $(\pi^e)$  and interest income taxation. Despite this, when Tanzi adjusted the coefficient of  $(\pi_t^e)$  for income taxes and reestimated Fisher's model, with the index of economic activity built in, the regression coefficients and the adjusted  $(R^2)$  decreased noticeably. Therefore, he concluded that although the investor could see through the veil of monetary illusion, they have suffered from "the fiscal illusion". The absence of the Darby effect however, does not indicate any irrationality for the investors.

#### Limitations of the Index of Economic Activity

Tanzi's model is limited because it does not include those assets which by nature are tax exempt, and those assets with tax-exemption advantages. Most importantly, the model does not capture other investment

alternatives available for investors (see Ezrati, 1982). Consequently, the model is not in line with the way sophisticated asset markets function. The overall equilibrium in the asset markets is reached when the after-tax, inflation-adjusted, and risk-adjusted rates are equal in all the markets. Otherwise, investors reallocate funds from one market to another, until all asset markets are clear simultaneously.

Tanzi expects that the coefficient of  $(\pi_t^e)$  is close to unity if all the different aspects of inflationary expectations are reasonably captured in the nominal interest rates. However, his regression results which use different ways to form  $(\pi_t^e)$ , show that the coefficients on  $(\pi_t^e)$  are less than unity. This may be because Tanzi does not have a mechanism by which he can capture the interaction effects of returns of different assets, after adjustments for taxes and inflation are made.

One possible way of broadening Tanzi's model is to include Mundell's effects into the model. According to Mundell (1963), an increase in the expected rate of inflation would decrease the real demand for money, which in turn would offset some of the upward pressure put on the nominal interest rate.

Empirical tests that failed to show the Darby effect, including Tanzi's, are reduced-form models. Therefore, to be able to test Darby's proposition, the structural model must be set up, and tested empirically (see Melvin, 1982). In a general equilibrium model in which commodity, labor, and money markets are considered simultaneously, the regression coefficient of  $(\pi_t^e)$  is not just  $1/1-T$ , but rather  $E+1/1-T$ , (where  $E$  depends on the interest elasticity of demand for money). As such, given a

fair amount of elasticity of demand for money, the coefficient of  $(\pi_t^e)$  must be less than  $1/1-T$ , unless the demand for money is interest inelastic - which would be a very special case. The existence of the Mundell effect in the model requires a coefficient of  $(\pi_t^e)$  less than  $1/(1-T)$ , but this does not refute the Darby effect.

Tests of the reduced form models (i.e., Melvin's) also create identification problems, which in turn imply that the process of going from the reduced to the structural model is complex. Each of these identification processes need their own statistical treatment and procedure. Generally, there are three different types of identification problems. 1. The just-identified, which refers to a case where there is a unique solution in which estimated coefficients are transformed from the reduced model to the structural model. In this case, the classical (OLS) estimation generates inconsistency in the coefficients estimated. Thus, the indirect least squares method can be used in order to avoid such biased estimations (see Teh-Wei Hu, 1973). 2. The over-identified, which is a case where there are many solutions when coefficients are transformed from the reduced to the structural model. In such an environment, the (OLS) does not provide unbiased regression results, simply because the error terms are not exogenous. That is, the error term is correlated with the explanatory variables in the model. Hence, the two-stage least squares method (2SLS) (as was used by Lahiri and Tanzi) is the only procedure with consistent results. In a more severe case in which the error terms in a general simultaneous structural model without a reduced form are correlated, the (2SLS)

cannot cure the problem. Thus, a three-stage least squares method (3SLS), or the full information maximum likelihood procedure (FIMLP), is needed. Although (FIMLP) is efficient and provides unbiased results much of the time, it is very complex. So the (3SLS) is a very common way of unbiasedly estimating the structural model. 3. The under-identified, which refers to the nonexistence of any solution after the regression coefficients are transformed from the reduced to the structural model (see Zellner and Theil 1962).

Finally, unlike Tanzi's belief in regards to the impact of the business cycle on the real interest rate through expected inflation, the level of economic activity affects  $(i_t)$  and  $(r_t^e)$  via unanticipated inflation (see Silver and Fackler, 1982). Silver and Fackler attempted to disentangle the dual impact of business cycles on  $(\pi_t^e)$  and  $(r_t)$ , by empirically testing the exact Fisherian formula, i.e., equation (2.2)' with the interaction term  $(r_t^e \pi_t^e)$ . Through the same procedure used by Tanzi (1980), they related realized real rate of interest  $(r_t^r)$  to the level of economic activity. However, as was mentioned before, if income fluctuates, then  $(\pi_t^a)$  is equal to  $(\pi_t^e)$  plus a measure  $(\mu > 0)$  of the index of economic activity  $(G_t)$ . In essence,  $(\mu G_t)$ , can be thought of as the discrepancy between  $(\pi_t^a)$  and  $(\pi_t^e)$ , or the unanticipated inflation rate.

The problem with such empirical results is three-fold. One, is that the interaction term in most studies completed in this area, turns out to be small and close to zero. Secondly, the common interaction function problem is applied, i.e., it creates an environment in which the regres-

sion coefficient of an explanatory variable depends on the level of the other variable in the interaction term. The coefficient on the interaction term is of no particular interest to a researcher, and might make the statistical results subject to doubtful conclusions. Thirdly, the test is not in line with the Fisherian hypothesis, because the relevant variable in Fisher's formula is the expected nominal/real interest rate, as opposed to realized real interest rate (see Tanzi 1982).

#### Ayanian's Model

Ayanian's (1983) model is basically Darby's model which was shown by equation (2-5)', and can be rewritten as;

$$i_t = \frac{(r_t^e + \pi_t^e)}{(1-T)} \quad (2-10)$$

Ayanian believes that the empirical research which deals with the Darby effect fails to show such effects - not because the Darby effect doesn't exist, but rather because incorrect data have been used in these studies. By incorrect data here, he meant that since  $(r_t^e)$  and  $(\pi_t^e)$  are both expected values of the real after-tax interest rate and the inflation rate respectively, by definition then, the actual data on  $(r_t^e)$  and  $(\pi_t^e)$  are not available. Therefore, any proxies for  $(r_t^e)$  and  $(\pi_t^e)$  will necessarily involve some approximation and thereby measurement errors.

To avoid the data problem, according to Ayanian, although the data on  $(r_t^e)$  and  $(\pi_t^e)$  are not accurate, the sum  $(r_t^e + \pi_t^e)$  can be viewed as yields on a tax-exempt bond ( $i_\chi$ ) - such as municipal bonds. Then, the test of the Darby effect is to regress yields of a taxable bond ( $i_T$ ) on a

tax exempt bond of the same maturity and risk whose returns are determined by  $i_{\chi} = (r_t^{'e} + \pi_t^e)$ .

As will be seen in Section III, Ayanian tested the Darby effect under a very extreme assumption. That is, regardless of the state of expectations associated with  $(r_t^{'e})$  and  $(\pi_t^e)$ , they jointly determine returns on the prime grade municipal bonds. However, he claims that assuming such a proxy for a tax-exempt bond does not mean that  $(r_t^{'e})$  is constant, or that  $(\pi_t^e)$  has been measured without error.

Ayanian's model over the sample period 1952-1979 shows very strong evidence of the Darby effect, and the compensation of the lenders for taxes on interest income, as well as the expected inflation rate. He tested the model for two sub periods - (1952-1965) and (1966-1979), both of which showed the existence of the Darby effect.

Ayanian's model is faced with two sets of problems - one is empirical and the second is theoretical. Empirical drawbacks remain to be seen in Section III, but the theoretical unsoundness is as follows. Glancing back at the equality between  $i_{\chi} = r_t^{'e} + \pi_t^e$ , it seems as though there are no measurement errors at all. Ayanian mentioned that this idea should not be inferred, but saying so is one thing, while giving it econometric content is quite another. First of all, most published data, especially if aggregated from monthly to quarterly data for example, contain measurement errors. Secondly, the expected inflation rate  $(\pi_t^e)$ , by nature, is associated with errors, which creates errors of a different type in  $(i_{\chi})$ .

If Ayanian's approximation is trivially rewritten differently, then;

$$r_t^{'e} = i_{\chi} - \pi_t^e \quad (2-11)$$

$(i_{\chi})$  is known and could be observed in the bond market. Therefore, the original model whose expected value is shown in equation (2-11), i.e., the actual after-tax real interest rate ( $r'_t$ ) is

$$r'_t = i_{\chi} - \pi_t \quad (2-12)$$

subtracting equation (2-11) from equation (2-12) results in

$$r'_t - r_t^{e} = -(\pi_t - \pi_t^e) \quad (2-13)$$

Equation (2-13) indicates that the positive error in estimating inflation under rational expectations, generates negative errors in the forecasted after-tax real interest rates. As such, when  $(i_{\chi})$  is regressed on  $(i_T)$ , the error term in  $(\pi_t^e)$  is correlated with one of the explanatory variables, and the OLS procedure gives inconsistent results. Consistent estimates require the use of a two-stage least squares method that corrects the regression coefficient for such a correlation of an explanatory variable with the error term.

Ayanian's (1983) proxy remains valid if there is either full-indexation, or  $(i_{\chi})$  stands for returns on a fully liquid and safe asset, i.e., money. In equation (2-13),  $(r'_t - r_t^e)$  is the unanticipated component of the real interest rate  $(r_t^{\text{un}})$ , and  $(\pi_t - \pi_t^e)$  denotes the unanticipated portion of the inflation rate  $(\pi_t^{\text{un}})$ . A more simplified version of equation (2-13) is

$$r_t^{\text{un}} = -\pi_t^{\text{un}} \quad (2-14)$$

For the case of full-indexation,  $\pi_t^{\text{un}} = 0$  implies that there is no error, and  $r_t^e = r'_t$ . If the asset is money whose nominal returns are zero, then the real return on money is the negative of the rate of inflation. By the



same token, the expected real return on money is the negative of the expected rate of inflation, and it follows that equation (2-14) holds.

#### Concluding Remarks

The empirical tests of the Darby effect that have evolved over the past decade can be classified as follows. Class 1 - The class of studies that assume the crucial relationship between the after-tax nominal interest rate ( $i_t$ ) and the expected rate of inflation ( $\pi_t^e$ ), depends on the risk associated with the return on physical capital, relative to yields on financial assets. Also, in these studies the utilization rate of capital, along with the liquidity effect, are among the determinants of ( $i_t$ ). Class 2 - The empirical works in which the absence of the Darby effect exists, due to the absence of the index of economic activity in Darby's model. The most common tactic of such studies is to use directly observed price expectations from survey data and the past rates of inflation, in order to generate different proxies for ( $\pi_t^e$ ). Class 3 - Studies which reject Class (2), because the model is a partial model and is incapable of reflecting the way complex asset markets function. Furthermore, the model has not incorporated assets that are tax-exempt, as well as adjusted after-tax, inflation-adjusted returns of alternative assets. In all of these studies, different yields of short-run bonds, such as Treasury bills and commercial paper, have been used for ( $i_t$ ). Correspondingly, the expected after-tax real interest rate is affected through different monetary, income, and expectational channels.

Class 4 - Finally, there is a study that has regarded inflation expectations as an exogenous parameter. That is, although data on  $(\pi_t^e)$  and the expected after-tax real interest rate  $(r_t'^e)$  are difficult to obtain, the sum of  $(r_t'^e + \pi_t^e)$  can be viewed as the yield on a tax-exempt bond - for example, municipal bonds. Therefore, the Darby effect can be tested in the context of a model, in which yields on a taxable bond (Treasury bonds) are regressed on yields of a tax-exempt bond (prime grade municipal bonds). In essence, this approach believes that regardless of how expectations are formed,  $(r_t'^e + \pi_t^e)$  jointly determines the returns on a tax-exempt bond, so that the Darby effect can be tested. Thus, if the coefficient of  $(\pi_t^e)$  is greater than unity - it indicates that indeed, investors have been compensated not just for expected inflation, but also some for the taxes on the returns of their assets.

## SECTION III. EMPIRICAL RESULTS

## Recapitulation of Ayanian's Model

As witnessed by equation (2-10), Ayanian (1983) empirically tested for the existence of the Darby effect by regressing quarterly averaged yields of one-year Treasury bills ( $i_T$ ) on one-year prime grade municipal bonds ( $i_X$ ). The sample period ran from 1952-1979 inclusively, and the data for the two variables ( $i_T$  and  $i_X$ ) were drawn from the Federal Reserve Bulletin and Salomon Brother's Bond Market Round-up, respectively. The regression results are (see Ayanian, page 763)

$$i_T = 0.158 + 1.63i_X; R^2 = 0.94 \quad (3-1)$$

(0.114) (.038)

The numbers in parentheses show the standard errors. From equation (3-1), Ayanian (1983) reported that the Darby effect existed over the sample period, because the regression coefficient of ( $i_X$ ) is greater than unity. That is, every one percentage point increase in ( $i_X$ ) is associated with a 1.63 increase in ( $i_T$ ), and this satisfactorily compensates lenders for the marginal tax rate ( $T = 38.7$ ).

Ayanian (1983) tested equation (3-1) for two subperiods, 1952-1965 and 1966-1979, both of which showed that the regression coefficient of ( $i_X$ ) is greater than unity. Furthermore, the Darby effect for the subperiods implied marginal tax rates of 43.5 and 36.3 respectively.

## Reestimating Ayanian's Model

Equation (3-1) has been reestimated for the sample period 1952-1979, by using the same data source for ( $i_T$ ) and ( $i_X$ ) as Ayanian's. Regression

results of the ordinary least squares method (OLS) are

$$i_T = 0.137 + 1.63i_X \quad (3-2)$$

(0.110) (0.037)

$$D.W. = 0.40 \quad R^2 = 0.94 \quad \rho = 0.82.$$

The numbers in parentheses are the standard errors, ( $\rho$ ) is the first order autocorrelation coefficient, and D.W. is the Durbin-Watson statistic.

In comparing Ayanian's results reported in equation (3-1) with equation (3-2), very minor differences are noticed. However, with the large value of ( $\rho$ ) and the small D.W., the presence of residual autocorrelated regression disturbance terms is apparent.

To account for serially correlated disturbances, let us redefine Ayanian's equation number (7) as reported on page 763, by adding an error term ( $U_t$ ).

$$i_T = i_X / (1-T) + U_t \quad (3-3)$$

( $U_t$ ) is serially correlated, i.e., the current value of ( $U_t$ ) is a fraction of the error term of the past period ( $U_{t-1}$ ), plus a classical error term ( $\varepsilon_t$ ).

$$U_t = \rho U_{t-1} + \varepsilon_t \quad (3-4)$$

( $\rho$ ) is the autocorrelation coefficient in which  $-1 < \rho < 1$ , and ( $\varepsilon_t$ ) is a white noise error term. Hence, the expected value of ( $\varepsilon_t$ ) is zero -  $E(\varepsilon_t) = 0$ . Also, for ( $U_t$ ) the following assumptions are fulfilled: 1. the variance  $\delta^2(U_t)$  is constant -  $V(U_t) = E(U_t - EU_t)^2 = \delta^2$ , and ( $U_t$ ) is homoscedastic. 2. ( $U_t$ ) is uncorrelated with ( $i_X$ ). 3. ( $U_t$ ) is normally distributed with zero population mean and constant variance.

Substituting equation (3-4) into equation (3-3) will result in

$$i_T = \frac{i_X}{(1-T)} + \rho U_{t-1} + \varepsilon_t \quad (3-5)$$

Equation (3-5) is the corrected version of equation number (7), reported by Ayanian. It is corrected in the sense that the regression coefficient of equation (3-5) is corrected for autocorrelation, by using the Cochrane-Orcutt Correction Procedure (CORC).

Generally, there are two different ways by which the regression coefficients can be corrected for pure autocorrelation. The first procedure is known as Generalized Least Squares (GLS), or is sometimes referred to as the Aitken estimator. In applying (GLS), it is assumed that  $(\rho)$  has already been estimated in equation (3-5). To simplify (GLS) procedure, assume in equation (3-3) that  $i_T = Y_t$ ,  $i_X = X_t$ , and  $1/(1-T) = b$ , therefore,

$$Y_t = bX_t + U_t \quad (3-6)$$

where  $(b)$  is the regression coefficient. The one-period lagged value of equation (3-6) in which both sides are multiplied by  $(\rho)$ , is

$$\rho Y_{t-1} = b\rho X_{t-1} + \rho U_{t-1} \quad (3-7)$$

subtract equation (3-7) from equation (3-6)

$$Y_t - \rho Y_{t-1} = b(X_t - \rho X_{t-1}) + U_t - \rho U_{t-1}. \quad (3-8)$$

From equation (3-4),  $U_t - \rho U_{t-1} = \varepsilon_t$ , therefore

$$Y_t - \rho Y_{t-1} = b(X_t - \rho X_{t-1}) + \varepsilon_t \quad (3-9)$$

The autocorrelation ( $U_t$ ) is eliminated and (OLS) can be applied to equation (3-9). The way (GLS) is designed, requires the inclusion of the intercept throughout the transformation procedure. That is, in equation

(3-9), the intercept is also multiplied by  $(1-\rho)$  and the regression coefficient (b) has the minimum variance.

The (CORC) is a different procedure than (GLS), because it estimates  $(\rho)$  and then goes through the correction process. The (CORC) by using (OLS), computes the residual  $(U_t)$  in equation (3-3), then estimates  $(\rho)$  by regressing  $(U_t)$  on its one-period lagged value, as shown in equation (3-4). The estimated value of  $(\rho)$  will be applied to the transformed equation (3-9), while the intercept is added. If the procedure ends here, then a two-stage (CORC) is employed. However, most computer packages do not stop at the second stage, but rather they obtain another estimate of the residual in the original model, along with a new estimate of  $(\rho)$  by following again the same procedure explained before. Subsequently, the newly estimated  $(\rho)$  will be applied to the newly transformed model. The iterative process<sup>1</sup> will come to an end when the newly estimated  $(\rho)$

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<sup>1</sup>In essence, (CORC) through its iteration procedure, minimizes the sum of squared errors  $\sum_{i=1}^N e_i^2 = \sum_{i=1}^N [(i_{T,t} - \hat{\rho}i_{T,t-1}) - \hat{a}(1-\rho) - \hat{b}(i_{X,t} - \rho i_{X,t-1})]^2$

where  $\hat{\rho}$  = estimate of  $\rho$

$i_{T,t-1}$  = lagged value of  $i_T$

$\hat{a}$  = estimate of the intercept

$\hat{b}$  = estimate of the regression coefficient

$i_{X,t-1}$  = lagged value of  $i_X$

For more information, see Johnston 1972, pp. 243-266.

differs from its preceding one by 0.0010 (see Cassidy 1981).

The corrected regression results for Ayanian's model are

$$i_T = 0.850 + 1.41 i_X \quad \text{D.W.} = 2.07 \quad (3-10)$$

(0.342) (0.072)

the resulting iteration procedure is also reported in the following table.

Table 3-1. Iteration results

| Iteration | (ρ) Coefficient |
|-----------|-----------------|
| 1         | 0.0000          |
| 2         | 0.8211          |
| 3         | 0.8774          |
| 4         | 0.8969          |
| 5         | 0.9036          |
| 6         | 0.9059          |
| 7         | 0.9066          |

A brief comparison between equations (3-1) and (3-10) indicates that the intercept in the corrected model is slightly larger than zero and significant at the five percent significance level. Also, the coefficient on ( $i_X$ ) is smaller than the uncorrected model, nevertheless, it is still significantly greater than unity. The significance of the intercept is due to the (CORC) corrective procedure, that adds some of the autocorrelated residuals to the intercept.

Ayanian's estimated regression coefficients for the two subperiods, along with our reestimated models and corrected models, are reported in Table 3-2;

Table 3-2. Sub-periods regression coefficients (Ayanian 1983, p. 764)

| Sample Period | Ayanian's Results                          | Reestimated Model                           | Corrected Model                            |
|---------------|--|---|--|
| 1952-1965     | $i_T = -0.07 + 1.77i_X$<br>(0.118) (0.096) | $i_T = -0.066 + 1.74i_X$<br>(0.119) (0.069) | $i_T = 0.641 + 1.33i_X$<br>(0.220) (0.102) |
|               | $R^2 = 0.92$                               | $R^2 = 0.92$ $\rho = 0.55$                  | D.W. = 1.84                                |
| 1966-1979     | $i_T = 0.36 + 1.57i_X$<br>(0.402)(0.104)   | $i_T = 0.263 + 1.60i_X$<br>(0.393) (0.102)  | $i_T = 1.03 + 1.44i_X$<br>(0.587)(0.109)   |
|               | $R^2 = 0.81$                               | $R^2 = 0.81$ $\rho = 0.86$                  | D.W. = 2.08                                |

As before, the numbers in parentheses show the standard errors. For the second half of the sample period, the autocorrelation problem seems to be more serious than for the first half. This is inferred by a larger sized ( $\rho$ ), and the size of the intercept in the corrected model. However, for all subperiod models that are corrected for residual autocorrelation, the coefficient of ( $i_X$ ), which is an indicator of the Darby effect, is significantly greater than one.

#### Monthly Observations

To investigate how serious residual autocorrelation is, Ayanian's model has been reestimated by using monthly observations on ( $i_T$ ) and ( $i_X$ ). The rationale behind substituting monthly observations for quarterly data is that if autocorrelation is monthly, then the impact on quarterly data is a lot weaker, i.e., quarterly averaged data mask pure actual residual autocorrelation.



Regression results of equation (3-1) with monthly data, turn out to be

$$i_T = 0.181 + 1.62i_X \quad (3-11)$$

(0.068) (0.023)

$$D.W. = 0.509 \quad R^2 = 0.93 \quad \rho = 0.748.$$

The iteration results for ( $\rho$ ) are also reported as follows.

Table 3-3. Iteration results

| Iteration | ( $\rho$ ) Coefficient |
|-----------|------------------------|
| 1         | 0.0000                 |
| 2         | 0.7480                 |
| 3         | 0.8816                 |
| 4         | 0.9738                 |
| 5         | 0.9914                 |
| 6         | 0.9906                 |

The coefficient on ( $i_X$ ) is very close to Ayanian's equation (3-1). However, the intercept now is significantly greater than zero, and the large value of ( $\rho$ ) indicates that the regression coefficient of equation (3-11) might have been affected by autocorrelation. Therefore, equation (3-11) has been corrected for possible serial correlation, and the results are

$$i_T = 3.59 + 0.545i_X \quad (3-12)$$

(1.28) (0.060)

$$D.W. = 1.89.$$

The regression coefficient of ( $i_X$ ) is significantly less than one, compared to equation (3-11), as well as in Ayanian's model described in equation (3-1). In terms of the Darby effect, the corrected coefficient indicates that a one percentage point increase in the nominal yields on one-year tax exempt bonds, corresponds to a 0.545 percentage point increase in

the nominal yields of one-year taxable bonds. This by no means compensates the lenders for inflation and taxes on their nominal returns. The intercept of equation (3-12), unlike equation (3-11), is larger and statistically significant, which is due to the (CORC) procedure.

Ayanian's model has been tested for the two sub-periods, both with the (OLS) and (CORC), and the results are reported in Table 3-4.

Table 3-4. Regression results of monthly observations

| Sample Period | OLS Estimates   | CORC Estimates  |
|---------------|---|---|
| 1952-1965     | $i_T = -0.040 + 1.75i_x$ <p style="text-align: center;">(0.074)<sup>a</sup>(0.043)</p> $R^2 = 0.90 \quad \rho = 0.71$ | $i_T = 1.54 + 0.807i_x$ <p style="text-align: center;">(0.332)(0.106)<sup>x</sup></p> $D.W. = 1.91$ |
| 1966-1979     | $i_T = 0.465 + 1.54i_x$ <p style="text-align: center;">(0.236) (0.061)<sup>x</sup></p> $R^2 = 0.79 \quad \rho = 0.76$ | $i_T = 4.69 + 0.516i_x$ <p style="text-align: center;">(0.907)(0.082)<sup>x</sup></p> $D.W. = 1.88$ |

<sup>a</sup>The numbers in parentheses are the standard errors.

The (OLS) results are fairly comparable to those of Ayanian's with quarterly observations, but the corrected regression coefficients of ( $i_x$ ) are significantly less than unity at the five percent significance level. The latter half of the sample period is particularly associated with a stronger residual autocorrelation, as was true with the quarterly data.

## Extension of Ayanian's Model

Equation (3-1) has been used to estimate the coefficient of ( $i_X$ ) for the sample period 1979 through August 1983 with monthly data, and 1979 through June 1983 with quarterly data inclusive. Both quarterly and monthly observations of ( $i_T$ ) and ( $i_X$ ) from the same data source mentioned before, have been utilized. The (OLS) and (CORC) adjusted regression results are shown in Table 3-5.

Table 3-5. Regression results of monthly observations

| Sample Period                                     | OLS Estimates   | CORC Estimates   |
|---|---|--|
| Monthly Data:<br>(1979-August, 1983<br>inclusive) | $i_T = 2.61 + 1.28i_X$<br>(0.759) <sup>a</sup> (0.116) <sup>X</sup> | $i_T = 6.08 + 0.733i_X$<br>(1.21) (0.177) <sup>X</sup> |
|   | $R^2 = 0.69$ $\rho = 0.61$  | D.W. = 1.72  |
| Quarterly Data:<br>(1979-June, 1983<br>inclusive) | $i_T = 2.21 + 1.35i_X$<br>(1.35) (0.206) <sup>X</sup>               | $i_T = 2.32 + 1.33i_X$<br>(1.57) (0.241) <sup>X</sup>  |
|   | $R^2 = 0.72$ $\rho = 0.23$  | D.W. = 1.93  |

<sup>a</sup>The numbers in parentheses are the standard errors.

The regression coefficient on  $i_X$  in the corrected model with monthly observations is consistent with the above CORC estimated coefficients for the two subperiods. Moreover, the extended models with quarterly data, both OLS and CORC, produce coefficients on  $i_X$  which are not significantly greater than one.

Comparably, quarterly data show smaller serial correlation in the residuals of the model. The corrected model with quarterly data indicates that for a one percentage point increase in ( $i_X$ ), the rate on a taxable bond such as Treasury bills, increases by 1.33 percentage points. This is enough to make up for both expected inflation and taxes on returns of bonds, so that the Darby effect is apparent. Nevertheless, the same conclusion cannot be derived using monthly data. Therefore, over the sample period, for a one percent increase in ( $i_X$ ), ( $i_T$ ) rose by only 0.733 percent (significantly less than one) which is not enough to compensate the lenders for expected inflation and income taxes.

Ayanian's Model from the 50s to the 80s

Equation (3-1) has been tested over the sample period 1952-1983, and the following coefficients for both quarterly and monthly data are obtained.

Table 3-6. Regression results

| Sample Period                                     | OLS Estimates  | CORC Estimates  |
|---|--|---|
| Monthly Data:<br>(1952-August, 1983<br>inclusive) | $i_T = 0.202 + 1.61i_X$<br>(0.069) <sup>a</sup> (0.018) <sup>x</sup> | $i_T = 3.53 + 0.581i_X$<br>(0.93) (0.063) <sup>x</sup>  |
|   | $R^2 = 0.95$ $\rho = 0.66$   | D.W. = 1.81   |
| Quarterly Data:<br>(1952-June, 1983<br>inclusive) | $i_T = 0.159 + 1.63i_X$<br>(0.107) (0.029) <sup>x</sup>              | $i_T = 0.356 + 1.56i_X$<br>(0.197) (0.053) <sup>x</sup> |
|   | $R^2 = 0.96$ $\rho = 0.55$   | D.W. = 2.13   |

<sup>a</sup>The numbers in parentheses are the standard errors.

Statistical inferences reported in Table 3-6 are very akin to those in the sample period 1952-1979. However, for both monthly and quarterly observations in uncorrected models, the autocorrelation coefficient ( $\rho$ ) is smaller than before. The smaller estimated value of ( $\rho$ ) obtained here is probably more nearly correct because there is now a larger sample size, which provides more degrees of freedom. Furthermore, for the entire period with monthly data, CORC estimates a regression coefficient on  $i_x$  which is significantly less than unity.

The interesting results are a comparison between  $\rho$ 's over two different sample periods, and are summarized in the following table.

Table 3-7. Autocorrelation Coefficients

| Sample Period | Monthly Data  | Quarterly Data |
|---------------|---------------|----------------|
| 1952-1979     | $\rho = 0.74$ | $\rho = 0.82$  |
| 1952-1983     | $\rho = 0.66$ | $\rho = 0.55$  |

When the sample size is increased, while at the same time the time interval is lengthened from monthly to quarterly observations, then  $\rho$  is decreased - which is a sign of detecting pure autocorrelation. The opposite conclusion was reached over 1952-1979, when monthly data were substi-

tuted for quarterly data. That is, autocorrelation became more severe as the time interval lengthened. This suggests that autocorrelation was impure, and could mean either a misspecified model, or the absence of statistically important explanatory variables in the model.

The existence of pure autocorrelation after 1979 could also be the result of many institutional changes that the U.S.A. experienced in the early '80s. Interest rate gyrations of the early 1980s were the outcome of changes in the monetary sector of the U.S.A. If the interest rate shocks "linger over" for a while, then pure autocorrelation picked up by the model is inevitable (see Arak and Guentner, 1983).

## SECTION IV. SUMMARY AND CONCLUSIONS

Ayanian's reported estimates exaggerate the Darby effect over the sample period 1952-1979. The taxable yields of one-year Treasury bills, regressed on the corrected coefficient of yields of tax-exempt prime grade municipal bonds of the same maturity and risk, is much smaller than Ayanian's. Thus, an unusually high Darby coefficient is subject to residual autocorrelation which has not been taken into account by Ayanian's model.

The problem of autocorrelation is very common with time series data, and could be either of pure, or impure types (see Cassidy, 1981). Pure autocorrelation occurs when the error term of the model is correlated with its own lagged values. Therefore, a random disturbance in the model "lingers over" for several time periods. Whereas, in the case of impure autocorrelation, the model has either been misspecified, or has not included some important explanatory variables.

If the source of autocorrelation is pure, then the regression coefficients estimated by (OLS) are unbiased,<sup>1</sup> as long as there is no lagged dependent variable. The existence of the lagged dependent variable causes the correlation between regressors and the error term, which makes (OLS) estimates biased. In such a case, substituting the two-stage least squares method for (OLS) is essential. However, as long as pure autocor-

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<sup>1</sup>However, the regression coefficients do not have the minimum variance.

relation is correctly detected, then the Generalized Least Squares (GLS) or (CORC) can be used in order to correct the regression coefficients for residual autocorrelation.

For impure autocorrelation, although the fix-up techniques detect the problem, unbiased coefficients require the specification of the correct model, or the inclusion of the variables that have been left out. The missing variables artificially deflate the estimates of the variance of other coefficients of explanatory variables, because the absent variables now become part of the error term in the model.

When regression models are estimated over several different time intervals for the data, for example monthly and quarterly, the D.W. detects more positive pure autocorrelation as the time length is shortened. This in turn creates a dilemma, because the more frequent the number of observations are (say weekly data), the more accurate the statistical results are (larger sample size). However, the problem of pure autocorrelation will be more serious. Likewise, the less frequent the number of observations are (say yearly data), the less accurate the results are (a smaller sample size). But autocorrelation will be less serious. Of course with weekly data, the autocorrelation is so severe that the disturbance term fully dominates the deterministic part of the model.

Impure autocorrelation is detected under two conditions. 1 - If the autocorrelation coefficient ( $\rho$ ) increases as the time period is lengthened (for example from monthly to quarterly observations), then the source of autocorrelation is impure. 2 - A negative value of ( $\rho$ ) indicates that the



serially correlated errors alter signs from one period to another. This is contrary to the idea of pure autocorrelation, because the disturbances are supposed to "linger over". Thus, the autocorrelation must be of an impure type.

It is rather a difficult problem to know which time intervals for the data should be employed? For some variables, the answer is straight forward - for example, if the model is concerned with the relationship between a consumer's income and the purchasing of an automobile, then the yearly data fit best in the model. However, for other less obvious cases, the answer depends on the dynamics of the model that should be specified correctly. On these grounds, for the shorter time span for aggregation, a more dynamic model that is capable of dealing with longer lags is recommended. Whereas with longer time intervals, a more static model fits the data best (see Cassidy 1981).

The obviously impure autocorrelation that is detected in Ayanian's model, indicates that the model is either misspecified (has wrong functional form), or has not incorporated enough explanatory variables. Confronted with such problems, one may question the validity of the Darby effect in the context of his model.

## REFERENCES

- Arak, M. and K. Guentner. "The Market For Tax-Exempt Issues: Why Are the Yields So High?" National Tax Journal 36, No. 2 (1983): 145-61.
- Ayanian, Robert. "Expectations, Taxes, and Interest: The Search for the Darby Effect." American Economic Review 73, No. 4 (September 1983): 762-765.
- Barro, J. Robert. Macroeconomics. New York: John Wiley and Sons, Inc., 1983.
- Carlson, John A. "Expected Inflation and Interest Rates." Economic Inquiry 17 (October 1979): 597-608.
- Cassidy, J. Henry. Using Econometrics - A Beginner's Guide. Reston, Virginia: Reston Publishing Company, 1981.
- Darby, Michael R. "The Financial and Tax Effects of Monetary Policy on Interest Rates." Economic Inquiry 13 (June 1975): 266-76.
- Dornbusch, Rudiger, and Stanley Fischer. Macroeconomics. New York: McGraw-Hill, 1978.
- Elliott, Jan W. "Measuring the Expected Real Rate of Interest: An Exploration of Macroeconomic Alternatives." American Economic Review 67 (June 1977): 429-44.
- Ezrati, Milton J. "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates: Comment." American Economic Review 72 (1982): 854-57.
- Fama, Eugene F. "Interest Rates and Inflation: The Message in the Entrails" American Economic Review 67 (June 1977): 487-96.
- Feldstein, Martin. "Inflation, Income Taxes, and the Rate of Interest: A Theoretical Analysis." American Economic Review 66 (December 1976): 809-20.
- Fisher, Irving. The Theory of Interest. New York: The Macmillan Company, 1930.
- Gibson, William E. "Interest Rates and Inflationary Expectations: New Evidence." American Economic Review 62, No. 5 (December 1972): 854-65.
- Harris, Laurence. Monetary Theory. New York: McGraw-Hill, Inc., 1981.

- Hu, Teh-Wei. Econometrics, An Introductory Analysis. Baltimore: University Park Press, 1973.
- Johnston, J. Econometrics Methods. Second edition. New York: McGraw-Hill, 1972.
- Lahiri, Kajal. "Inflationary Expectations: Their Formation and Interest Rate Effects." American Economic Review 66 (March 1976): 124-31.
- Melvin, Michael. "Expected Inflation, Taxation, and Interest Rates: The Delusion of Fiscal Illusion." American Economic Review 72, No. 4 (September 1982): 841-45.
- Mundell, Robert A. "Inflation and Real Interest." Journal of Political Economy 71 (June 1963): 280-83.
- Silver, J. Lew and James S. Fackler. "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates: Comment." American Economic Review 72 (September 1982): 72, 858-59.
- Tanzi, Vito. "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates." American Economic Review 70 (March 1980): 12-21.
- Tanzi, Vito. "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates: Reply." American Economic Review 72 (September 1982): 860-63.
- Zellner and Theil. "Three Stage Least Squares: Simultaneous Estimation of Simultaneous Equations." Econometrica 30 (January 1962): 54-78.

## GENERAL CONCLUSIONS

According to the empirical results in part one, an unanticipated increase in the rate of growth of the money supply ( $UM_t$ ) creates a dominant liquidity effect. Therefore, an increase in ( $UM_t$ ) decreases nominal interest rates in different bond markets. The interest rate models which were tested empirically, showed a significant price expectational effect associated with an anticipated increase in the money supply growth rate ( $AM_t$ ). Thus, an increase in ( $AM_t$ ) increases interest rates, and anticipated changes in the money supply growth rate, at least with quarterly time series data, do matter. Furthermore, given the efficiency for bond markets, the significant liquidity effects of ( $UM_t$ ) and price expectational effects of ( $AM_t$ ) depend in part upon the choice of a particular time series data.

The empirical results of part two cast doubt about the existence of the Darby effect over 1952-1979. Also, it implies that the reported Darby effect is subject to question because of impure autocorrelation. That is, when the time span is lengthened from monthly to quarterly, the autocorrelation coefficient grows in size. Therefore, with the existence of impure autocorrelation, the model either has a wrong functional form, or it has excluded important explanatory variables.

When the model is extended to the '80s, the statistical results show a smaller sign of residual autocorrelation, when the time interval is lengthened. In essence, in the '80s the model detects pure autocorrelation. However, the model extended and corrected for pure autocorrelation shows

that the regression coefficient on the expected rate of inflation, which is a measure of the Darby effect, is less than unity. For the Darby effect to be apparent, the coefficient on  $(\pi_t^e)$  would need to be significantly greater than one.

In summary, the two essays in this study have mainly dealt with the response of interest rates to anticipated and unanticipated money supply growth rates, as well as the rate of inflation. In general, interest rates were found to respond to these factors in the direction predicted by economic theory, but the magnitude of the response did not seem to be as large as expected theoretically.